Competitive Quote Flipping and Trade Clusters

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ABSTRACT

We model the decision to exhaust depth by high speed traders, which either flips the best bid or ask quote to the opposite side or widens the spread. Such events are common and often revert to the previous best quote levels. Consistent with the model, such quote flipping results in large trade clusters at the competitive equilibrium. Using the order book for the S&P E-mini futures contract, we document quote changes and find on average 78% of these revert to previous best quote levels within 3 seconds and about one-third of these events are isolated over a 40 millisecond window from other quote changes. Trade clusters are found before a quote change. Specifically, 18.1% of volume arises within 2 milliseconds of an isolated, reverting change in quotes. Empirically, this model explains at least as much quote change activity as does a liquidity replacement view.

Keywords: Order book, Depth, Quote flippers, Clustering, E-mini futures

JEL classification: G10, G13

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I. INTRODUCTION

Two phenomena that are challenging to explain in microstructure models are frequent quote changes and the clustering of trades. Figure 1 illustrates these events using a sample of 250 quote changes for the S&P E-mini futures contract traded on August 6, 2014 beginning at approximately 10:00 A.M (E.T.). There are 20,894 contracts traded over the 326 seconds in this sample. The figure shows three levels of activity at various intervals in the image, as if the bid- and ask-quotes differ by two price ticks with trades occurring at the midpoint. In fact, this is predominately a one-tick market where both the best bid and ask prices change frequently, about once every 1.3 seconds. Trades occur at all three levels over the figure. Trades cluster near these quote changes because 66% of the volume is within 5 milliseconds of these quote changes. Importantly, these quote changes return to their previous levels about 78% of the time.[1] Our question here is why do trades cluster so often near quote changes that revert to previous levels?

A simple answer is that there is a loss of liquidity around trade clusters, so quotes change to attract new orders from liquidity providers, who then frequently return quotes to previous levels if there is no information event (Harris (2003)). The problem with this view is that these depth depletions and quote reversions tend to repeat sequentially many times as shown Figure 1. Thus, while liquidity providers may be slow to react, they will surely understand the monitoring cost of repeated order submissions and consider increasing the size of their quote restoring limit orders making these reverting quote changes less common. Also, this observation begs the question of why trades cluster at all.

[1] The findings in Roberts (2018) for quote changes in the WTI crude oil futures market show similar reversion results for quote changes.

Another possibility is that random or possibly strategic cancellations may accelerate the depletion of depth caused by trade clusters. These cancellations may act to create quote changes and simultaneously avoid adverse selection. On this view, liquidity providers act quickly to cancel orders, but then change their minds to restore quotes to previous levels. Although this is a sensible strategy to avoid adverse selection costs from informed traders, it seems inconsistent with quote reversion rates of 78% and repeated sequences of quote changes and reversions. Also, if you think order flows are becoming toxic, then set your limit orders away from the best quotes instead of cancelling and then re-submitting them at the previous levels.

Informed trading is also a possibility for why quotes change. Such situations may create a race to access liquidity in the market (Budish, Cramton and Shim (2015)), which can generate trade clusters and accelerate the move to a new fundamental price. In such cases, traders may target stale quotes and extinguish depth quickly (Foucault, Hombert, and Roşu (2016), Aquilina, Budish, and O'Neill (2020)).[2] Consistent with this view, however, is that the market is not expected to return to previous quote levels without additional information to reinforce such a move. In our data, we measure the likelihood of informed trading based on the occurrence of a two-tick or more directional move in quotes. In our sample, this event occurs about 12% of the time after a quote change, so most of the time informed trading is not an explanation for quote changes or trade clusters.

This paper suggests a different view. Specifically, we develop a model in which trade clusters and *reverting* quote changes are due to heterogeneous speed technologies across participants. Specifically, some participants are faster than others and these high frequency traders (HFTs) may use speed to implement a simple, opportunistic strategy. The strategy we

[2] An early example of this phenomena arose when the Nasdaq instituted a firm quote rule for its Small Order Execution System (SOES) in the early 1990s giving rise to so-called SOES bandits (Harris and Schultz (1997, 1998)).

model creates quote changes followed by quote reversions as there is no information effect, and when HFTs are in competition with each other also results in trade clusters.

Specifically, we show that fast traders act to deplete depth at the margin so that they may gain from temporary changes in the best quote levels.[3] We call these traders "quote flippers" because they consider whether to flip a quote that has diminished depth before (slower) liquidity suppliers can respond to replenish depth at the best quotes.[4] These quote flippers are faster than liquidity suppliers even if they both receive market signals at the same time, which is a point embedded in the model by Budish, Cramton and Shim (2015).[5] This environment suggests a marketplace with differing trader speeds, such as modelled by van Kervel (2015) and Baldauf and Mollner (2020). Empirical support for this environment is found by many researchers, including Hasbrouck and Saar (2013), SEC study (2014), Brogaard, et al. (2015) and Fishe, Haynes, and Onur (2019).

Our approach implicitly treats quote flipping behavior as a mixture of liquidity replacement and informed trading because quote flippers may appear as if they are informed so new quotes may be justified, but the truly informed will discover this falsehood and trade or change resting limit orders to bring quotes back to their previous levels, making it appear as if volume was uninformed and the quote change was a liquidity event. Quote flippers may gain only when they can expect to exit at a better price. Thus, they must place opposite side liquidating orders on the

[3] Roberts (2018) shows that activity after a quote change (e.g., cancellations and new order submissions) provides valuable information to market participants, so quote flippers may benefit from such information even when they cannot exit a flipped position.

[4] Of the three dominant HFT strategies identified by Boehmer, Li, and Saar (2018), the one modelled here corresponds to "short-horizon directional speculation."

[5] Budish, Cramton and Shim (2015, p. 1560) state: "…our model clarifies that in the continuous market fast traders can earn a rent even from information that they observe at exactly the same time as other fast traders…" Hasbrouck (2018, p. 618) also makes a similar observation as it relates to forecasting: "…although prediction is usually viewed as forecasting what has yet to occur, it is functionally equivalent to more timely knowledge of what has in fact already happened."

book and expect that market orders will arrive to match such exit orders.[6] This approach is similar to the Edgeworth cycle story of Hasbrouck (2018), because our participants can reach a competitive equilibrium in which expected profits are zero from quote flipping, but still if all participants operated with the same technology they could avoid any excess costs from the temporary price flips.[7]

Empirical support for our model is found by examining participant order flows around quote changes in the S&P E-mini futures contract, which is based on the S&P 500 stock index. As noted, these data show that 78% of quote changes revert to previous best quote levels, 10% maintain the new levels with the spread the same as before the change, and about 12% continue to other higher (or lower) quote levels. Of those that revert to previous best quote levels, between 22-25% of the initial quote changes arise because cancellations exhausted standing depth.[8] Our interest here is in the remaining cases caused by trades because many of these report transactions at the new (temporary) quotes before a reversion. Thus, providing quote flippers an exit opportunity.

Consistent with previous research we build on the order book environment developed by Glosten (1994) and Sandås (2001) to characterize the behavior of quote flippers. In this environment, limit order participants are uninformed about the future fundamental value but have common knowledge of the current value and how informed trading is expected to change value. When (possibly) informed market orders arrive after the posting of limit orders, the relative size of buy and sell volumes establishes the effects of any informed trading on the

[6] Quote flippers may also gain if they exit at any subsequent better prices because they inadvertently anticipated informed trading. We abstract from this possibility to focus on exits from standing orders placed on the book at the next-best quotes.
[7] Noel (2012) makes an analogous claim about Edgeworth cycles in gasoline prices by showing the consumers could reduce costs with a strategy that purchased at the trough of the Edgeworth cycle.
[8] See Van Ness, Van Ness, and Watson (2015) for empirical evidence on the effects of cancelled orders in security markets.

fundamental price. If such trading depletes depth on one side at the previous best quote, then a new best quote arises and the limit order book may repopulate around this new price level. This is efficient as liquidity providers perceive that the fundamental value has changed.

However, if a quote flipper acts to deplete depth at the best quote, the same book repopulation may occur as only informed participants may correct this mistake. This misunderstanding by liquidity providers gives quote flippers an opportunity to gain if they can exit before the mistake is corrected.[9] Because market orders may be informed or uninformed, quote flippers depend on the arrival of uninformed market orders to make their exit. We show how this environment leads to a range of depth that is profitable to flip.[10] If depth at the best quote falls into this range during the market-order step in the model, then quote flippers may act in mass to deplete the remaining depth and flip the best quote, which generates a trade cluster. Our model weighs the probability that this strategy is successful against the likelihood that market order flow is insufficient for an exit, so quotes revert and flippers hold a possibly losing position.

Because there may be positive expected profits from quote flipping and entry is limited only by investments in a speedy technology, a competitive solution should arise in which quote flippers' gains are driven to zero. At the model's competitive solution, when depth reaches a tipping point many quote flippers enter trades and quickly exhaust depth. Thus, competitive quote flipping implies a large clustering of trades before a quote change.

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[9] This claim derives from Allen and Gale (1992) who show that an uninformed participant can gain if other participants attach a positive probability that the price change is from informed trading.

[10] This result follows from the literature on speculative attacks. In currency markets, when central bank holdings of foreign reserves drop below a target level and the exchange rate peg is not an equilibrium, then speculators attack the peg to try to deplete all of the foreign currency reserves, forcing central banks to devalue the exchange rate (Connolly and Taylor (1984)). A similar situation may arise when a government tries to fix the price of gold (Salant and Henderson (1978)). In market microstructure, the depth at the best quotes acts to defend these quote levels against a speculative attack.

Numerous researchers have documented trade clusters or periods of higher trade intensity in trade and quote data (e.g., Dufour and Engle (2000), Sarkar and Schwartz (2006), and Bowsher (2007)).[11] Theoretical explanations for such results include the incentive of informed traders to act quickly on their information (Riordan, Storkenmaier, Wagener, and Zhang (2013), Brogaard, Hendershott, and Riordan (2014), Menkveld and Zoican (2017)), arbitrage opportunities that increase trade intensity (Foucault, Kozhan, and Tham (2017)), and tradebots reacting to each other at nanosecond frequencies (Menkveld (2017)). By examining isolated quote changes, we show that a disproportionate percentage of trade volume appears in a 2 millisecond (ms) interval before a quote change that reverts to previous levels. Thus, the competitive reactions of quote flippers provide a different explanation for trade clusters, one not dependent on news signals or price-dependent arbitrage opportunities.

The sample data analyzed here are daily files from Vertex Analytics, which are organized message feeds from the Commodity Mercantile Exchange (CME). Message data are also used by Aquilina, Budish, and O'Neill (2020) to study the market-wide costs of latency arbitrage trades. These message files show all updates to the CME's electronic order book for the ten best quote levels. These data also designate the initiator of a trade. We study trading in the E-mini futures contract beginning on August 1, 2014 and ending September 30, 2014, a total of 42 trading days. These quotes and trades are organized by the most active expiration by volume, starting with September 2014 and switching to December 2014. The E-mini contract is often the first or second most active futures contract and includes thousands of individual participants, making it

[11] Trade clusters include the special case of price clusters, which arise when some prices are preferred to others. Explanations for price clusters focus on the price attractiveness of rounded numbers (Kandel, Sarig, and Wohl (2001); Ahn, Cai, and Cheung (2005)), a least-cost negotiation hypothesis (Harris (1991)), manipulation and collusion (Christie and Schultz (1994), and cultural factors (Brown, Chua, and Mitchell (2002)).

likely to contain different speed technologies and more likely that some participants could be misled by quote flipping activity versus contracts involving fewer traders.

We identify quote change events and classify a subset into instant and lagged events, where the first type arises because both the best ask and bid quotes change simultaneously, and the second type arises because one side responds with a lag. The data show that the instant-type flips, where an order remainder is left at the flipped quote, is the most common type. For both types of quote flips, we find significant support for the existence of quote flippers based on the how the frequency of quote flipping responds to our model parameters. However, we also find some support for the view that liquidity replenishment may be delayed causing a temporary change in quotes. Thus, our results suggest that quote flippers exist, but are not fully responsible for the subset of quote changes in our sample.

The sample data also show that a large portion of volume is clustered. We partition these data into quote change events that are isolated and grouped. Isolated events contain only one quote change centered in a 40ms window. Grouped quote change events occur close to each other in windows that begin and end with a 10ms gap. In total, 41.4% and 26% of all volume arises in isolated and grouped event windows, respectively. Clustering is evident because these windows account for only 7.9% of the sample clock time. Importantly, the isolated windows show that volume increases up to a quote change and that 18.1% of volume arises within 2ms of a quote change that reverts to the previous quote levels. Our model predicts this type of behavior when quote flippers are competitive.

This paper proceeds as follows. Section II discusses the modeling of the order book and how quote flippers may be introduced to affect the basic equilibrium. Section III describes the data, the incidence of quote changes, and the empirical strategy we develop to test for the quote flipping and liquidity replenishment views. Section IV provides our analyses and results. Finally, Section V offers a few conclusions.

II. MODEL

The static order book model of Glosten (1994) has been adapted to examine different microstructure effects including the depth of the order book (Sandås (2001)), the effects of fast and slow traders on competition between trading venues (van Kervel (2015)), and the incentive to cancel limit orders (Dahlström, Hagströmer and Nordén (2017)). Although Sandås (2001) rejects the basic model in a small sample of Swedish stocks, Beltran, Grammig, and Menkveld (2005) find that the environment is generally supported using the DAX30 stocks when conditioned on the information levels in each stock's trade flow. This finding suggests that the Glosten environment may be appropriate for our problem as we adjust the timing and information structure in the basic model to examine the effects of quote flippers.

The basic model assumes discrete prices, time priority for orders placed at the same price, and a matching engine without dealers to create trades when market orders arise. Inventory effects are not considered, so participants trade for market making, liquidity, or informed purposes. Private values are omitted but may be introduced for liquidity suppliers without meaningfully changing the basic results.[12] Traders are parsed between liquidity providers who submit limit orders and liquidity takers who submit market orders. The endogenous decision of the optimal order type is not considered (see Parlour and Seppi (2008) for a survey). The order type choice may be less of concern here because the parsing of traders separates the potentially informed from those who supply liquidity based on an initial common understanding of

[12] Dahlström, Hagströmer and Nordén (2017) suggest that allowing order processing costs to vary across liquidity suppliers introduces private values. If such costs are independent of market-order volumes, then this variation results in a marginal profit condition that depends in the distribution of order processing costs.

fundamental value. Also, the relatively short time windows in this study may make choices between limit and market orders less important.[13]

A. Basic Model with Modified Liquidity Demand

We begin with the model structure developed by Sandås (2001) and Dahlström, et al. (2017). There are two steps in deriving the order book at period *t*. In the first step immediately prior to *t*, limit orders are submitted. These orders depend on the expected behavior of market orders at *t*, the fundamental value of the risky asset, X_t, a common order processing cost, γ, and the impact of trading volume on the fundamental value. Limit order traders are assumed to know the current fundamental value when placing their orders.

The fundamental value evolves as new information arrives, characterized by a random error term, such that next period's value is $X_{t+1} = X_t + d_{t+1}$. Liquidity-demanding traders may have information on the zero-mean innovation term, d_{t+1}. Following Foucault, Pagano, and Röell (2013, Ch. 4), liquidity suppliers know of such potentially informed traders and include a price impact function when forming expectations about next period's fundamental value:

$$E[X_{t+1}|X_t, m_t] = X_t + \alpha m_t, \quad (1)$$

where m_t represents signed market order flow and the price impact effect, α, is linear in the market order flow.[14] As market orders may be informed or uninformed, α is not a pure information or adverse selection effect, but an implicitly weighted coefficient based the relative population of informed and uninformed market-order traders.

[13] Order type choices may be affected by several variables, including the patience allowed by a trader's strategy, and the actions of other traders in equilibrium. See Parlour (1998), Foucault, Kadan and Kandel (2005), Rosu (2009), and in a multi-venue setting, Cont and Kukanov (2017).
[14] The effect of nonlinear price impact functions is explored by Farmer and Skouras (2011).

The second step arises at period t when liquidity demanders arrive with market orders. The maintained assumption is that the exponential distribution characterizes market order flows. We modify this assumption to allow two draws from the exponential distribution to characterize liquidity demand. This allows us to consider the effects of quote flipping before the second draw. It also reflects a modeling choice that allows liquidity demanders to act more quickly than liquidity suppliers can change orders or otherwise react to incoming information. In effect, liquidity demanders are on average faster traders than liquidity suppliers, which is parallel to the approach of van Kervel (2015) in a multi-venue setting. This is the essential friction embedded in our model.

From the view of liquidity suppliers, the liquidity demand density is the convolution of two exponential random variables, where $m_t = m_{1,t} + m_{2,t}$, and $m_{1,t}$ and $m_{2,t}$ are each independent market order samples from an exponential distribution.[15] In the basic Sandås (2001) model a two-sided exponential distribution with parameters (φ_A, φ_B) represents the density of orders on the ask and bid sides, respectively. With the two-sample convolution, however, the density on each side depends on the combined outcome of the two order flows. We assume that $\varphi_A = \varphi_B = \varphi$ for expositional convenience. Thus, for the ask side cases where the two draws both produce buyers because $m_1 \geq 0\ and\ m_2 \geq 0$, the density is Erlang$(2, \varphi)$. For bid side cases where the two draws produce sellers because $-m_1 \geq 0\ and - m_2 \geq 0$, the density is also Erlang$(2, \varphi)$ with the functional form modified to reflect these negative values. For cases that mix these outcomes, $m_1 \geq 0\ and - m_2 \geq 0$ or $-m_1 \geq 0\ and\ m_2 \geq 0$, there may be either net positive outcomes (buyers) or net negative outcomes (sellers) with equal probability. These cases give

[15] When no confusion will arise, we drop the t subscript in the following notation.

rise to a two-sided Laplace(φ) distribution where each case may have positive or negative sums. The resulting convolution density on the ask- and bid-sides is given by:[16]

$$f(m) = \begin{cases} \frac{1}{4\varphi^2} m \exp\left(-\frac{m}{\varphi}\right) \; if \; m_1 \geq 0 \; and \; m_2 \geq 0 \; (orders \; trade \; at \; ask \; price) \\ \frac{1}{8\varphi^2} \exp\left(-\frac{m}{\varphi}\right) \; if \; m_1 \geq 0, -m_2 \geq 0 \; and \; m \geq 0 \; (orders \; trade \; at \; ask \; price) \\ \frac{1}{8\varphi^2} \exp\left(-\frac{m}{\varphi}\right) \; if \; -m_1 \geq 0, m_2 \geq 0 \; and \; m \geq 0 \; (orders \; trade \; at \; ask \; price) \\ \\ \frac{1}{8\varphi^2} \exp\left(\frac{m}{\varphi}\right) \; if \; m_1 \geq 0, -m_2 \geq 0 \; and - m \geq 0 \; (orders \; trade \; at \; bid \; price) \\ \frac{1}{8\varphi^2} \exp\left(\frac{m}{\varphi}\right) \; if - m_1 \geq 0, m_2 \geq 0 \; and - m \geq 0 \; (orders \; trade \; at \; bid \; price) \\ -\frac{1}{4\varphi^2} m \exp\left(\frac{m}{\varphi}\right) \; if \; -m_1 \geq 0 \; and - m_2 \geq 0 \; (orders \; trade \; at \; bid \; price) \end{cases} \tag{2}$$

The first three terms in expression (2) establish the density for situations in which the order flow is net positive; the last three terms are for net negative cases. The net positive and negative cases in the convolution density are adjusted by one-half in these expressions to indicate that buyside and sellside orders are equally likely to arise at period *t*. These fractions are allocated as $\left\{\frac{1}{4}, \frac{1}{8}, \frac{1}{8}; \frac{1}{8}, \frac{1}{8}, \frac{1}{4}\right\}$ between the ask- and bid-side terms. The parameter φ is the expected size of market orders when m_1 or m_2 is drawn from the exponential distribution.

Liquidity providers build the order book by evaluating expected profits from limit orders at each price level. On the ask side of the book, expected profits for liquidity providers placing an order are given by:

$$E\left[\pi_{i,t} \mid m \geq \vec{Q}_i\right] = P_i - \gamma - E\left[X_{t+1} \mid m \geq \vec{Q}_i\right] \tag{3}$$

[16] The convolution structure assumes that the draws are from the same distribution function for buys and sells, respectively. Changing this assumption gives rise to the same basic results but allows analyses of issues such as traders anticipating large order flows. These concerns are beyond the scope of this study.

where P_i is the limit order price for an order at the i^{th} price level on the book, $\vec{Q}_i$ denotes the (combined) buyside market order volume necessary to execute an order given the current state of the book at this price level, and the order processing cost is assessed on each transaction. The index *i* marks the price grid sequence; it takes values, 1,2,3,…,N to denote the grid steps on the ask side and -1,-2,-3,…,-N on the bid side, with 1 and -1 indicating the current best ask and bid quotes, respectively. The notation for $\vec{Q}_i$ is from Dahlström, et al. (2017), who relate cancellations to the size of orders in front of a given order (including that order) and to orders behind the given order. The right pointing arrow refers to orders in front. In the discussion below, we must know the size of quote flippers orders, so we will interpret $\vec{Q}_i$ as orders on the book with higher priority than the quote flippers' order size. Note that the expected profit function on the bid side follows an analogous form.

Using the density in (2), the expected profit at the best ask quote for the marginal or last order included in the volume measured by $\vec{Q}_1$ may be expressed as follows:

$$E\left[\pi_{1,t}^{A} \middle| \, m \geq \vec{Q}_1\right] = \frac{1}{4\varphi}\left[(P_1 - \gamma - X_t)(2\varphi + \vec{Q}_1) - \alpha(2\varphi(\vec{Q}_1 + \varphi) + \frac{\vec{Q}_1^2}{2})\right] e^{-\vec{Q}_1/\varphi} \qquad (4)$$

where the current fundamental value, X_t, the distributional parameter, φ, and the information effect, α, are known to all participants when limit orders are submitted. The last term within the brackets represents adverse selection (or information) costs based on buyside volume being greater than or equal to $\vec{Q}_1$.

Equilibrium arises for liquidity providers under the assumption that participants are competing for limit order priority at the various grid prices, which results in zero profits for the marginal limit order at each price level. Effectively, the term within brackets in equation (4) is set equal to zero to solve for depth at a given price level. Extending the Sandås (2001) results, the zero-profit condition implies that depth at the *kth* ask (P_k) or bid (P_{-k}) price satisfies:

$$(P_k - \gamma - X_t)(2\varphi + \textstyle\sum_{i=1}^{k} \vec{Q}_i) - \alpha(2\varphi(\sum_{i=1}^{k} \vec{Q}_i + \varphi) + \frac{\left(\sum_{i=1}^{k} \vec{Q}_i\right)^2}{2}) = 0 \tag{5}$$

$$(X_t - \gamma - P_{-k})(2\varphi + \textstyle\sum_{i=-1}^{-k} \vec{Q}_i) - \alpha(2\varphi(\sum_{i=-1}^{-k} \vec{Q}_i + \varphi) + \frac{\left(\sum_{i=-1}^{-k} \vec{Q}_i\right)^2}{2}) = 0 \tag{6}$$

A positive solution for depth derives from the quadratic formula. Specifically, the competitive levels of depth at the best ask ($\vec{Q}_1^A$) and best bid ($\vec{Q}_{-1}^B$) price levels are given by:

$$\vec{Q}_1^A = \frac{2(P_1 - \gamma - X_t)}{\alpha} - 2\varphi \tag{7}$$

$$\vec{Q}_{-1}^B = \frac{2(X_t - \gamma - P_{-1})}{\alpha} - 2\varphi \tag{8}$$

These equations are similar to the results derived by Sandås (2001) except that depth is compounded twice to account for the two draws from the exponential distribution. The solutions derived from equations (7) and (8) may be substituted back into the zero-profit expressions to solve recursively for the optimal depth levels on the entire price grid.

B. Quote Flippers Enter the Market

To examine how quote flippers may exploit this order book model, we insert an additional action within period *t*. After the first sample is taken from the distribution for market orders, a quote flipper considers whether to enter the market to flip a quote on either the best ask or bid side. Then the second sample is taken from the market order distribution. To liquidity providers, this appears as if a market order exhausted depth at one of the best quotes. From the point of view of liquidity suppliers, there is no observable difference between liquidity takers and quote flippers as all they see is a stream of market orders. Thus, we do not change the structure of the liquidity suppliers' optimization problem. As such, this is not a model with complete rational expectations.

The sequence of steps in the quote flipper model is illustrated in Figure 2. First, liquidity providers place quotes on the order book up to the point at which marginal profits are zero. Second, market order traders arrive at period *t*, and some of these participants may be informed. Quote flippers arrive after this first round of market orders. If the remaining depth at the best quotes is such that quote flippers expect positive profits from "attacking" the remaining depth, then these participants trade to remove the remaining depth, possibly leaving an open order that changes an ask to a new best bid or vice versa. Quote flippers then place an exit limit order equal to the size traded at the new, best quote. Lastly, the second round of market orders arrive, and the quote flippers' orders are traded, or they are left with inventory as the next period's true fundamental price is revealed. The process then repeats itself.

In this environment, quote flippers act with marketable limit orders to remove the best quote, say on the ask side (P_1) for the purpose of discussion, to establish the next-best ask as a new best quote. In effect, quote flippers are attempting to deceive liquidity providers and takers into thinking that P_2 is the new P_1, or that that P_1 is the new P_{-1} if the quote flipper leaves unexecuted order size on the book. Thus, the best ask and (possibly) bid prices have shifted up the price grid. The effects are that the (weighted) midpoint of the bid-ask spread is distorted from its fundamental value. We say this because in a single-draw version of this model by Dahlström, et al. (2017), a depth-weighted calculation using the best quotes provides an estimate of the fundamental asset value, such that $P_1 > X_t > P_{-1}$. If other participants were to make such a calculation, then quote flippers' actions (on the ask side) would imply a higher fundamental value, which might have the effect of encouraging more buy orders. Appendix A2 provides a similar result for this model.

It is not reasonable for participants to be mistaken for long, so we consider quote flipping as an ephemeral action, which lasts only until the end of the next round of market orders as some of those participants are informed—or hold signals that are informed in aggregate—about the true fundamental value in period *t+1*.[17] If enough market orders arrive to remove the quote flippers' exit order (placed at price P_2 after the resting depth of $\vec{Q}_2^A$ on the ask side), then the action yields profits. If not, then market orders act to restore the best quotes and the quote flipper holds a possibly losing position.

The above discussion outlines the structure of the quote flipping model. The analytical details are shown by the quote flipper's expression for expected profits. We derive this expression from a flip on the ask side, defined conditional on the realization of m_1 after the first draw of market orders. For a quote flip on the ask side (bid side is analogous), the expected profits are given by:

$$E\left[\pi_f \mid m_1\right] = \left(\vec{Q}_1^A - m_1\right)\{(P_2 - 2\gamma - P_1)prob\left(m_2 \geq \vec{Q}_2^A + \vec{Q}_1^A - m_1\right) +$$

$$(X_t + \alpha(m_1 + m_2) - \gamma - P_1)prob\left(m_2 \leq \vec{Q}_2^A + \vec{Q}_1^A - m_1\right)\}$$

$$= \frac{(\vec{Q}_1^A - m_1)}{2}\Big\{\int_{\vec{Q}_2^A + \vec{Q}_1^A - m_1}^{\infty}(P_2 - 2\gamma - P_1)\tfrac{1}{\varphi}e^{-m_2}\,dm_2$$

$$+ \int_0^{\vec{Q}_2^A + \vec{Q}_1^A - m_1}(X_t + \alpha(m_1 + m_2) - \gamma - P_1)\tfrac{1}{\varphi}e^{-m_2/\varphi}\,dm_2$$

$$+ \int_{-\infty}^{0}(X_t + \alpha(m_1 + m_2) - \gamma - P_1)\tfrac{1}{\varphi}e^{m_2/\varphi}\,dm_2\Big\} \qquad (9)$$

where $(P_2 - 2\gamma - P_1)$ represents the net gains per contract flipped after a successful exit, $(X_t + \alpha(m_1 + m_2) - \gamma - P_1)$ represents *potential* net losses if the exit order does not execute after

[17] Glosten (1994, p.1150) concurs with this assumption when he states, "However, the bluffer should expect that both information, and the arrival of other traders will tend to reverse the effects of the bluff. This is because the bluffer knows that the market's expectations have been artificially pushed up, and, hence, public announcements will, on average, provide correcting information."

being placed on the back of the book after $\vec{Q}_2^A$ contracts, which were optimally submitted by liquidity providers in the first step of the model, and $\left(\vec{Q}_1^A - m_1\right)$ is the quantity of contracts flipped. The probability density terms combine the likelihood of both net positive and net negative results for m_2 in the second draw from the distribution of market orders.

The second term in equation (9) represents "potential" net losses because $X_t - \gamma - P_1 < 0$. However, the sign of $(X_t + \alpha(m_1 + m_2) - \gamma - P_1)$ depends on the information effect. The magnitude of the information effect, $\alpha(m_1 + m_2)$, is partly known because m_1 is observed by the quote flipper. The expected size of market orders in the second draw without a successful exit is given by, $E\left[m_2 | m_2 \leq \vec{Q}_2^A + \vec{Q}_1^A - m_1\right] = -\left(\varphi + \vec{Q}_2^A + \vec{Q}_1^A - m_1\right) e^{-\frac{\vec{Q}_2^A + \vec{Q}_1^A - m_1}{\varphi}} < 0$. If the market order draw, m_1, is sufficiently positive, the potential net losses from flipping may be small or even offer net gains, which would then encourage more trades until depth is exhausted (see below).

Quote flipping is a viable strategy if the model admits parameter values that imply $E\left[\pi_f \mid m_1\right] \geq 0$. To determine if this situation arises, we investigate the properties of $E\left[\pi_f \mid m_1\right]$ across the range of m_1. Appendix A2 develops these basic results. Unfortunately, there is no closed form solution to this problem for either a competitive result, $E\left[\pi_f \mid m_1\right] = 0$, or for a maximum. Thus, we consider the profile of $E\left[\pi_f \mid m_1\right]$ and the effects of key model parameters by simulation using data from Sandås (2001) and the E-mini futures contract to initialize these simulations.

C. Comparative Statics of the Quote Flipper Model

To simulate expected profits, we parameterize equation (9) as follows: $P_1 = \$12.50$, $P_2 = \$25.00$, $X_t = \$6.25$, $\varphi = 10$, $\alpha = 0.10$, and $\gamma = 0$. The prices are chosen because the model only depends on the relative values of price variables; that is, $P_2 - P_1$ or $X_t - P_1$. The values here are selected based on specifications of the E-mini futures contract. The minimum tick size for this contract is \$12.50, so these choices imply the price grid is populated with limit orders using the minimum tick as price increments. The value of the fundamental price at period *t* is set to one-half the tick size implying a balanced weighting for depth in the formula derived by Dahlström, et al. (2017). The values for φ and α are selected from those estimated by Sandås (2001) using the range of values he provides in Table 5 for breakeven conditions, which he reports to be closer to the actual (data) schedules. Excluding one outlier (ticker SEB), he finds that estimates of the ask-side density parameter range from 4.3 to 19.9 (mean of 10.8) and the alpha range is from 0.004 to 0.21 (mean of 0.08). The initial values chosen here are both close to the averages of his estimates. The gamma parameter is set to zero as exchange fees are small relative to E-mini contract values.

Figure 3 shows the path of expected profits for three different values of α (0.05, 0.10, 0.15) as the flow of initial market orders (m_1) increases. In these plots, the depth at P_1 and P_2 changes based on the optimal solution to equation (5) at these price grid values. In each case the expected profit path ends at zero when $m_1 = \vec{Q}_1^A$. This figure shows that when market orders have a large impact on fundamental value (X_t), the expected profits from quote flipping are small and the range over which a quote flipper might profitably attack the remaining depth is relatively small. As the information/adverse selection impact of market orders diminishes, the plots show that the profit opportunities increase. This suggests that when order flows appear less toxic, quotes may be expected to change often as quote flippers deplete larger amounts of the remaining depth at the best quotes (Easley, López de Prado, and O'Hara, 2012). A possible offset to this prediction is that depth is decreased at the best quotes when alpha increases, so for a given market order flow, quotes may change more often independent of quote flipping behavior.

Figure 4 repeats the simulation in Figure 3 but with the density parameter (φ) changing. A larger φ parameter flattens the density but also represents a larger average market order flow. This would appear to encourage quote flippers. However, this also affects the liquidity providers' solution for depth, which decreases because they may be adversely selected when φ increases. The three plots in Figure 4 show that the effect on optimal depth at the best ask price leaves a smaller quantity remaining before a quote flipper expects to profit. This simulation implies that larger orders result in less depth at the best prices and smaller size attacks on the best quotes.

Note that in Figure 4 it appears that the three curves intersect when first crossing the zero axis. This is an illusion. As discussed below, this point represents a competitive solution that is affected to a small degree when φ changes (see Appendix A2).

The last simulation is shown in Figure 5. This figure plots the region where expected profits are positive based on the quantity flipped to understand how depth at the second-best ask affects quote flipping. The model in equation (9) places the quantity flipped at the back of the book for grid price P_2. However, the book may change with cancellations or modifications, or quote flippers may pre-position their orders based on an expectation that they may participate in quote flipping. For the latter case, these orders may move up the book as others cancel or modify their orders. These two plots show the existing case in which the flipped order is placed at the back of book and the other extreme in which the order is at the front of the book, ready to execute against an incoming market buy order. Not surprisingly, when the order is at the front of the book, expected profits from flipping increase. Thus, as the depth at the second-best level of quotes decreases, *ceteris paribus,* we expect more cases of quote flipping.

D. Competitive Equilibrium for Quote Flippers

As shown in the simulations, quote flippers may expect to profit from their strategy. However, a competitive environment is expected to drive expected profits to zero. Classically, entry occurs and creates more quote flipping until reaching an equilibrium. This occurs to the left of the profit maximum points shown in the above figures, at the value of m_1 where expected profits first reach zero. These possible equilibria points are labelled in Figure 3. Based on the simulations for Figures 3 and 4, the initial zero-profit position may range from 25% to 45% of the initial depth at the best quotes. As such, trade activity as depth is depleted is expected to accelerate markedly when removal becomes profitable. Given the size of the remaining depth to remove and a competitive response by fast traders, the outcome will be a large trade cluster (Sarkar and Schwartz (2006)). Trade clusters are predicted from models in which informed traders receive the same or similar signals. These quote flippers have no more information than liquidity providers, so their clustered response is not due to information signals but rather a technology advantage.

The final point to make about the competitive equilibrium is that the embedded speed advantage of quote flippers over liquidity providers may confuse liquidity providers as to the reasons for a quote flip. Both the quote flipper and liquidity provider observe m_1 and m_2 in sequence as this is public information. However, quote flippers are faster than liquidity providers so they can effectuate the removal of $\left(\vec{Q}_1^A - m_1\right)$ contracts before m_2 is known and before liquidity providers can react, which is only after both m_1 and m_2 are realized.[18] In effect, the quote removal may appear nearly simultaneous with the realization of m_1, blurring the distinction between the m_1 execution and the quote flippers' trades. If m_1 is sufficiently large

[18] A speed advantage could also be interpreted to imply that liquidity providers do not perceive the initial draw of market orders, m_1, as quickly as quote flippers.

relative to α, then the potential loss term, $(X_t + \alpha m_1 - \gamma - P_1)$, may be positive. In this circumstance, quote flippers improve the market by flipping the existing best ask price (P_1) to a new best bid price. Thus, a quantity flipped, $\vec{Q}_1^A - m_1 + \vec{q}_{-1}$, which leaves $\vec{q}_{-1}$ as the depth at the new best bid may be rationalized as an information effect by liquidity providers.[19] This possibility helps quote flippers disguise the true intent of their actions when the information effect is small, making the flip order arise for a strictly strategic purpose.

III. DATA

This section discusses the limit order book data and how it is filtered to examine quote-change events, quote patterns that appear in these data, and the empirical strategy used to test for quote flippers. We also provide summary statistics on volume, clustering, exit trades, and the explanatory variables used in our regressions. Appendix A1 provides additional technical details of the underlying data and processing/identification steps.

A. Message Data and Quote Patterns

Daily message files, for the most actively traded expiration, are downloaded from Vertex Analytics, which include all updates to the visible electronic limit order book for both sides of the E-mini market corresponding to the 10 best book levels and all signed trades.[20] Each message update to the limit order book contains information on the price level, number of contracts, and number of orders for a defined market side and book level. We sort by the group sequence

[19] An alternative view is that market order participants may require collateralized signals to validate the new quote(s). Quote flippers may collateralize their actions by leaving a residual quantity to display after removing the depth at either the best ask or bid. As there is a positive probability that these residual orders will execute without a profitable exit, the expected gains from quote flipping are diminished. Thus, a zero-profit equilibrium may be derived if the probability of a market order to buy or sell becomes conditional on whether sufficient depth exists at the new quotes. Then, the size of the remaining depth determines the expected losses necessary to offset the quote flippers expected gains.

[20] Vertex Analytics (https://vertex-analytics.com/) collects raw message data from the CME and breaks this information down into easily accessible tables.

identifier (ID) and then by the record sequence ID. We then convert the ordered book updates into a snapshot of the full visible limit order book sampled at the frequency of each update. As it is common to observe multiple book updates within the same message, we select the last book snapshot for each group sequence ID. Because we are focused on activity near the market, we limit our study to the reconstructed limit order book for the two best book levels on both sides of the market. Specific details of the book reconstruction are discussed in Appendix A1.

The reconstructed limit order book data show many patterns for changes in best quotes and bid-ask spreads, many of which are documented in Figure 6. Each image in Figure 6 represents a quote change *event*, characterized by a 1-tick bid-ask spread before and after a change in one or both best quotes. Most events can be defined as "standard" effects (Panels A and B) when there is a 1-tick movement at one or both best quotes. Panel C shows examples of other possible events, which we designate as "non-standard" effects. These non-standard effects make up a small share of total events (0.3%). For each event, we are document the actions in the book that cause the quotes to change. For example, for instant ask flips, it is common to see aggressive buying activity at the best ask which depletes the resting depth, and then a remaining portion of this aggressive activity rests on the book to establish a new best bid quote.

Panel A in Figure 6 illustrates common quote change effects when depth on the ask side of the market is depleted. This panel shows two cases in which both the best ask and bid quotes change and a third occurrence of when the best ask quote is removed but re-established without any observed change in the best bid quote. We distinguish the first two types of quote events in Panel A. The first is an instant flip as in the example above. The second is a "lagged" pattern so that there is a delay in one quote adjustment making the spread widen temporarily. Both cases may be examples of quote flipping. The third common observed event is defined as a 'hat' event and results when the best quote is removed and then re-established before the other side can respond. This third case is unique in that the spread changes, but the best quotes end up unchanged by the event.

To identify these quote events in the data, we select a single day and for each observation compute the bid-ask spread (BAS = best ask minus best bid). We then start from the first observation where the market provides a one-tick spread and move across the reconstructed limit order book to find changes in the bid-ask spread or the best quotes. If we find an observation with a BAS greater than one tick or a change in the best quotes, we mark it as the start of a quote change event. We make note of whether the start is due to a trade execution or a cancellation (see Appendix A1). We then continue moving through the book until there is an action that re-establishes the 1-tick bid-ask spread, which then defines the end of the first event. We continue processing observations in the rebuilt book until all quote change events are identified.

What happens following an event provides direction to our empirical approach. We characterize quote flip events in Panels A and B in Figure 6 by how they end, or the next quote change event. As the model considers quote flipping as ephemeral, our focus is on the quote change events that "revert" as demonstrated by an ask flip followed by bid flip (either instant or lagged). However, we recognize that quote changes may be more permanent and are "confirmed" by remaining unchanged as demonstrated by an ask flip followed by a hat event, or may represent information effects by "continuing" directionally as demonstrated by an ask flip followed by another ask flip. We use the next sequential quote event to classify these patterns. If the prior event indicates an instant or lagged quote event and if the next event causes quotes to revert to previous levels, either instantaneously or with a lag, then the prior event is a flip that reverted, or a "reversion", and is possibly due to quote flippers. If the next event continues the initial quote change with a same-side directional change, then this quote flip event appears as an information effect and is called a "continuation". Continuation events are considered "informed" and excluded from the regression analysis below.

Classifying the quote events that maintain the new quote levels is more problematic. Panels A and B in Figure 6 show two cases of hat events and Panel C shows examples of two 'non-standard' events. If the prior quote event is followed by a hat or other "non-standard" pattern, then we classify the initial flip event as a quote "confirmation", where the new quote levels are confirmed by the market until the next quote event. The data show that 98% of these confirmations are validated by hat events. In effect, liquidity is being provided at the new quote levels for the duration of the event. In our regression approach, we exclude quote events that are confirmed by the market, so that we only focus on reversions.

Table 1 shows a detailed summary of the types of quote events found in the E-mini futures data. These data show the initial direction of a quote change ("down", "up", or "not assigned"), the type of flip ("instant" or "lag") and the next future event, which allows us to define when an event ultimately concluded by reverting, confirming, or continuing directionally. These endings are marked with the opposite price direction (up-to-down or down-to-up) for the reverting cases, marked with the same price direction (up-to-up or down-to-down) for the continuing cases, and marked "not assigned" for the confirming cases where a hat or non-standard event ends the initial quote event. Counts and percentages are shown for events grouped by the initial price direction. These percentages show that down (up) flips revert 77.6% (77.7%) of the time. Thus implying that the majority of quote flips—both instant and lag—are temporary events. The instant-flip events are more common than the lag-flip events, implying that quote flippers may find it advantageous to leave a remaining quantity at the flipped quote level. Interestingly, the dominance of instant flips over lagged flips also holds for cases where the price level continues directionally, offering a view that liquidity providers may receive mixed signals when a remaining quantity is left on the book after a quote change. That is, some of the time the remaining quantity may act as collateral suggesting a signal of fundamental price direction, but most of the time the signal is wrong.

Table 1 also shows the average duration in seconds between these events as well as the standard deviation and the first three quartiles of the duration distribution. These data show that flip events that revert last on average less than three seconds with the median duration less than 80 milliseconds. These median data suggest that exit opportunities for quote flippers are likely to rely on algorithmic traders. The durations for continuing events are longer, ranging from 9.5 to 11.5 seconds for down-down events and 11.0 to 12.3 seconds for up-up events. These averages suggest that informed flows are not as quickly processed by the market as are uninformed (i.e., quote flips) flows that temporarily deplete liquidity.

B. Clustering of Trades

The model predicts that if quote flipping is competitive, a burst of trade activity will arise when depth is reduced to a critical threshold. This activity will result in the depletion of depth at a best quote and a new best quote on one or both sides of the market. To examine this question, we filter the quote events into three categories. The first category is called "isolated" events. These are quote flips that arise in relative isolation from other quote events. We use a 40ms window to isolate these events to study how trading changed before and after these events. In each case, there are no quote events in the 20ms before or after a quote event. The second category is called "grouped" events. This category includes all quote events that arise within 20ms proximity of each other. Each of these groups has a 10ms gap before the first event and after the last event. Between these gaps there are two or more quote events that are within 20ms of each other. The last category represents the remaining parts of the trading session not captured by quote events. This is a non-event group because trades may occur, but there are no quote events. Table 2 summarizes the volume found in these groups.

Volume in Table 2 is divided between the best quotes, second-best quotes, and trades deeper in the book when orders trade through both the best and second-best quotes. These data show that isolated events account for 41.4% of volume with most of this volume at the best quotes. Grouped and non-events account for 26.0% and 32.7% of volume, respectively. The table also shows the share of the day accounted for by these categories. Isolated events account for only 0.54% of the clock time in the sample, which is significant evidence of large trade clusters around these events. The grouped events represent 7.37% of clock time, which is also indicative of clustering. The remaining 92.1% of clock time accounts for just one-third of the volume. The small clock-time shares for isolated and grouped events relative to their volumes indicate the important role of trade clustering in price changes and eventual price discovery.

Table 3 investigates the clustering of trades for the isolated group in more detail. Volume is shown at the best quotes as in Table 2 with the information divided into those quote changes caused by trades or cancellations. Volume is also divided into 2ms intervals before and after the quote event, which begin and end at 10ms before and after an event.[21] These quote events are also classified as to the whether they reverted to the previous quote levels or continued directionally for at least one additional tick change in price. A residual category for non-standard quote changes called "other quote events" contains the remaining volume.

[21] We narrow the view of trades to a 15ms window to minimize the double counting of volume which arises when two isolated quotes changes are near to each other. That is, the second change is between 31ms and 45ms after the first change.

Table 3 shows rapid, nonlinear increases in volume as the market approaches a quote event. For trade-caused events that revert, the event volume of 6.1 million contracts exceeds the total volume in the previous five 2ms intervals. This also arises in the second column where the quote does not revert but continues directionally. In both cases, quote flippers may gain from their actions. This pattern is exactly what the model predicts to arise if quote flipping is competitive. That is, once depth is depleted to a certain level, competition between quote flippers causes a rapid removal of the best quote.

The table also shows the share of best quote volume for each 2ms interval and the event itself. Relative to all best quote volume, we see that 11.9% and 6.2% of contract volume arises at trade-caused events that either revert or continue directionally. This is fully 18.1% of best-quote trade volume occurring at the message frequency level, which is evidence of high frequency participants acting directionally during these events to create trade clusters.

C. Grouping Events into Intervals

The empirical strategy we follow to identify quote flipping is to divide the data into fixed time intervals and then count the instant and lagged flips found in these intervals. In this process we will remove the quote change events that were initiated by cancellations—approximately 22-25%—so only trade-caused events are included. We use count regression methods as discussed by Cameron and Trivedi (2013) to test the implications of our theory. The time intervals examined are 5-seconds and 60-seconds.[22] For the 42 trading days in our sample, there are 249,480 5-second intervals and 20,790 60-second intervals. However, we filter these to remove intervals that have less than 5 trades because many intervals have quote changes but no trades. These changes would clearly be for reasons unrelated to quote flipping. This filter generates

[22] We also considered 10-second and 30-second intervals, and although the coefficients changed, the signs and significance were similar to what we found for the 5- and 60-second intervals.

slightly less volume than reported in Table 2 (49,444,509 versus 52,675,928 contracts). This filter leaves 130,810 5-second intervals and 20,672 60-second intervals. Note that the quote-change event counts allow a flip to end in the next contiguous interval and to be included in the count for the current interval—but that flip is not counted twice.

Table 4 provides summary information on the volume in these intervals, flip counts by type, flips caused by cancelled orders, information on exit trades, and the explanatory variables used in the analyses. The 5-second intervals have an average volume of 370 contracts, about 14.5% of the average volume reported in the 60-second intervals. This suggests that volume is not uniformly distributed within a one-minute time frame. The comparison between the means and medians imply skewness in these volume distributions. The data report the side that initiated a trade, and we find that average buyer-initiated and seller-initiated volumes are about equal in these intervals. The medians here are nearly equal, too, but they also imply skewness. Such skewness is consistent with our finding that there are volume clusters across these intervals where episodic increases in trades arise.

The count data show that the average 5-second interval has about one instant quote flip on either the ask- or bid-side. The median in these cases is zero, suggesting that most 5-second intervals contain no instant quote flips; these are apparently intervals with sufficient market making liquidity at the current quotes. Of these, approximately 0.21 are due to cancellations or order removal from the best quotes, which are netted out in the regressions below. The average counts for lagged flips are less than one-half of those for instant flip, also with the median interval having zero counts.

The 60-second intervals report an average of 6.3 (2.5) instant (lagged) bid flips with these averages almost equal on the ask side. Similarly, the medians are 4.0 (1.0) instant (lagged) bid flips with the same observed on the ask side. Counts for cancel-caused flips are on average 1.4 (0.6) for instant (lagged) bid flips, with similar counts on the ask side but with all medians still zero. Thus, a 60-second window is likely long enough to observe a quote flip at least one-half of the time caused by a trade action, not a cancellation.

D. Exit Opportunities

For quote flipping to be a considered strategy, it must be possible for such participants to see viable exit opportunities. To this end, we computed the average number of trades at the exit side of a quote flip. This is the formerly second-best quote. These trade counts are computed for the flip event until its end. Table 4 shows that there are an average of 59 (46) trades at the new bid after an instant (lagged) bid flip. Similar averages are reported for ask side flips. These averages seem large but that is because the end of a flip event may not be a reversion to the old quotes. Instead, for this summary, we consider any ending type (revert, confirm, or continue) because a quote flipper only wants an exit execution, so it does not matter how the event ends. These data show that quote flippers may reason that there is a positive probability of an exit trade, which may increase if such participants pre-position their exit orders before a flip event.

E. Explanatory Variables

The explanatory variables are selected to conform to the theory of quote flipping. This theory suggests that the parameters, alpha and phi, as well as depth levels are important to whether it is profitable to flip a quote. Alpha is a parameter that converts signed order flows into fundamental price changes. To capture this idea, we draw upon the insight of Amihud (2002) who proposed an illiquidity measure based on the ratio of returns to dollar volume. As alpha is effectively a toxic order flow parameter, we consider the ratio of the quoted price range in the previous interval to trade volume as a proxy for how signed order flow may convert into fundamental price changes. The sample average for this estimator is 0.018 for the 5-second intervals, respectively, which conforms well to the estimates obtained by Sandås (2001) reported above. The average for the 60-second intervals is substantially less, suggesting that order flow impacts decrease as the measurement interval increases. Even so, some alpha estimates reported by Sandås (2001) are also relatively low.

The parameter phi is a measure of the expected order flow as determined by the market order distribution. This variable is estimated by the average trade size in the previous interval. The estimates shown here for the 5- and 60-second intervals are nearly equal, and at the lower end of those found by Sandås (2001). We use lagged/predetermined data for both alpha and phi variables to avoid an endogeneity problem with flips in the current interval. We expect that both alpha and phi variables will have a negative effect on quote flipping as illustrated by Figures 3 and 4. Intuitively, one may argue that average trade size (phi) should be positively correlated with flipped quotes as the larger the trades, *ceteris paribus*, then the greater the likelihood that depth is exhausted. Thus, a negative sign on the coefficient for this variable is an important signal that quote flippers are in this market.

We also include measures of book depth.[23] We evaluate the depth at the best ask (bid) at the beginning of an interval. If a large volume is necessary to exhaust this depth then exits will take longer, so this variable is expected to have a negative effect on flips. We also created the same variable for the second-best depth level. The larger is the depth at what will become an exit price, then the less incentive a trader has to create a quote flip without pre-positioning the exit order.

[23] We also consider a flow measure of depth by dividing beginning interval depth by lagged buyer-initiated (or seller-initiated) volume to measure the expected trade flow necessary to exhaust this quantity. These results confirmed the findings for our simple measures.

The 5-second and 60-second interval data show similar averages for beginning depth, in the mid-400s of contracts at the best quotes and approximately 1,000 contracts at the second-best quotes. The median depths are about two-thirds of the average for the best quotes and close to 85% of the second-best quote averages. The reduced skewness for the second-best quotes is consistent with the finding that most quote changes revert rather than continue to higher (or lower) quote levels. If the median depth was substantially less than the average depth at the second-best quotes, then there would be fewer intervals with substantial liquidity at these quotes, making price continuations more likely.

Lastly, our discussion of the quote-change theory admits the possibility that quote changes may be informed or liquidity events. We have removed potentially informed events by focusing on one-tick quote changes that revert to original levels.[24] However, if liquidity providers are slower than market order traders, then there may be many episodes in which depth is removed from a quote until liquidity providers are afforded enough time to restore the order book. This is effectively a story that volume of trading has exhausted depth. Thus, we include the volume of trading as well as trade count variables in the previous interval as a proxy for whether depth may be exhausted from on-going trades instead of from participants acting opportunistically to flip quotes.

IV. ANALYSIS

A. Count Regressions

Count regressions are estimated within a maximum likelihood framework, which requires a distributional assumption about the counts. The Poisson distribution is the usual assumption. We

[24] It is always possible that a one tick move may be due to information and a reversion may represent a second informed signal reversing the first signal. We acknowledge that there is no way to filter out these possibilities.

started the analyses with the Poisson assumption, but found that the data rejected the estimated model as the goodness-of-fit chi-squared test was statistically significant in all model specifications using the explanatory variables in Table 4. The count data exhibited strong evidence of over-dispersion; that is, the variance is greater than the mean, which are equal in the Poisson distribution. Cameron and Trivedi (2013) suggest that a negative binomial distribution, where the mean and variance are not necessarily equal, is a good alternative in the presence of over-dispersion. Thus, the analyses here assume the count data follow a negative binomial distribution.

Tables 5 and 6 present the maximum likelihood estimates of our count models. Table 5 shows results for the 5-second intervals and Table 6 shows results for 60-second intervals. The dependent variables are either the count of (net) instant flips in an interval or the count of (net) lagged flips, both of which exclude flips due to cancellations. In these tables, Panel A shows estimates when flips change the best bid and Panel B shows the same estimated models when the best ask is flipped. Each regression is estimated with daily effects as the counts vary significantly between days. We also provide Generalized Estimating Equation (GEE) results with the errors clustered by day to confirm the significance of the variable coefficients in the maximum likelihood regressions. The key variables from the quote flipping theory are the first four listed in each panel of these tables (excluding the intercept).

Focusing on Panel A in Table 5, the variables that are proxies for alpha and phi all show negative and significant coefficients for both instant and lagged flip counts as predicted by the theory. The Akaike Information Criterion (AIC), shown at the bottom of each estimated model, reinforces the inclusion of these variables in the model. Both the first-best and second-best bid depth variables have a negative and significant coefficient in these regressions. The coefficient on the second best depth is smaller so its effect on flipping is lower on a per contract basis.[25] The results for best ask flips in Panel B mirror those in Panel A with only slight variation in the magnitude of the coefficients.

Table 5 shows strong support for the variables that theory predicts to affect the expected profits from quote flipping based on the signs of these estimated coefficients. However, a competing theory is that trading exhausts liquidity and it takes liquidity providers some time to replenish depth. Thus, quote flipping may not be the only cause of the flips that we document. To consider this view, we included lagged total volume and lagged total trade counts in the count regressions. There are mixed results on the sign and significance of these variables. For Panel A, lagged volume is not significant, but lagged trade counts show a positive and significant effect on both instant and lagged flips. The GEE estimates confirm these findings. In Panel B, the ask side shows support for the liquidity claim for instant flips, but the sign on lagged volume is negative for lagged flips sending an ambiguous signal. On net, the trade count variable exhibits the correct positive, significant sign. Thus, it appears that it is the quantity of trades, more than the total volume that affects the likelihood of depth removal at the best quotes. Even though this is an unexpected result, we accept it as support for the liquidity replenishment theory.

The issue now is to decide the relative importance of the two theories represented by these estimates. Cameron and Trivedi (2013) offer an approach to measuring the (pseudo) r-squared in count regressions. The models estimated in Panels A and B in Table 5 show pseudo r-squared estimates. For the 5-second intervals, these variables explain relatively little of the variation in quote change counts. The daily fixed effects variables explain approximately 3%, those associated with the quote flipping theory explain another 5.0-6.8%, and those that we have

[25] It is also true that when simulating the effects of depth in Figure 5, the marginal effect on expected profits of moving to the front of the queue at the second-best quote was small compared how changes in alpha or phi affected expected profits.

connected to liquidity replenishment may explain another 3.0-4.0%. Thus, while a claim can be made that both quote flipping and liquidity replenishment theory have support, there appears to be slightly more power from the quote flipping variables even though there remains meaningful unexplained randomness in these 5-second interval data.

Table 6 reports estimates of the same count regression models using 60-second intervals. For the quote flipping variables, the results are very similar to Table 5 in terms of coefficient signs and significance. The coefficient magnitudes are now different, particularly for the alpha proxy variable. If this coefficient had scaled up proportionately over the interval, then its value would be between 5.6 and 9.7 using the GEE estimates in the two panels. Instead it is markedly larger suggesting that the longer interval may better capture the clustering nature of quote flipping. In contrast, the two depth variables show smaller coefficient magnitudes compared to Table 5. These coefficients are now all significant and all have the correct sign on lagged volume except for regressions using lagged flips on the ask side, which still shows a negative coefficient.

The findings in Table 6 also differ from those in Table 5 when measured by explanatory power. The explanatory power of these variables is increased meaningfully by using 60-second intervals to measure quote flips. The pseudo r-squared start at 12-14% using daily fixed effects, increase by 8-12% when the quote flipping variables are introduced and then by another 13-30% when the liquidity-replenishment variables are included. In the best-fit cases, about 45% of the variation in “instant” best bid or ask flips are explained by these variables. Compared to the 5-second interval results, there remains much less unexplained randomness in the flips counts after accounting for the influence of these variables. Thus, it appears that by capturing more clustered events in the same interval, both quote flipping and liquidity variables have improved explanatory power.

V. CONCLUSIONS

We provide a theory of quote flipping in which high speed participants use speed to deplete order book depth and flip a prevailing best quote to a second-best quote. Expected profits are positive from this strategy but may be driven to a competitive equilibrium by entry of other quote flippers with an equivalent technology. This theory explains the substantial number of one-tick quote changes that quickly revert to previous levels. The empirical evidence finds support for the view that some quote changes are linked to such behavior. The empirical analyses also finds that the standard liquidity replenishment view—that is, it takes time to re-establish order book depth—has support as well and may explain as much of the variation in the sample data as quote flipping.

Importantly, the evidence supports the view that competition in quote flipping results in clustered trades before quote changes, which differs from previous explanations of trade clusters—for example, informed traders capitalizing quickly on information. We conclude that quote flipping is an important element of quote changes and trade clusters, but there is still meaningful unexplained randomness that remains in the quote change data for the E-mini futures contract, supporting the assumption of idiosyncratic order flows found in many microstructure models.

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Appendix A1
Raw Message Data to Rebuild the Order Book

Market participants invest substantial resources to purchase and process CME message data in real-time. A raw message is a long string of tag and value pairs that defines the header of the message (e.g., message sequence number, date, arrival time, etc.) along with individual data blocks associated with specific instruments and quote updates.[26] We obtain these message data from Vertex Analytics, which has pre-processed the CME message stream to simplify the re-creation of the limit order book. Table A1 illustrates the raw Vertex data.

Table A1 – Raw Book Updates

Message ID	Update ID	Time	Side	Level	Price	Qty	NumOrders	Aggressor
M_1	U_1	T_1	Bid	1	50	30	5	
M_2	U_2	T_2	Ask	2	52	55	7	
M_3	U_{3a}	T_3	Bid*	1*	50	5		Sell
M_3	U_{3b}	T_3	Bid	1	50	25	4	
M_4	U_4	T_4	…	…	…	…	…	

*Information inferred from the price level and aggressor flag.

Each row in Table A1 indicates an update to information on the book at a particular time. For example, the first message (M_1) contains an update ID (U_1), a time (T_1), and specific information about the book. That information shows that the book is modified to reflect a best bid (side="bid" and level="1") at the price of 50 and a depth of 30 contracts composed of 5 orders. A single message such as M_1 can contain a data block related to a trade(s) at the best bid/ask, a

[26] Academic researchers can purchase historical raw CME message data here: https://www.cmegroup.com/market-data/datamine-historical-data/#marketDepth.

data block removing the best bid/ask, a data block updating depth at the N^{th} best bid/ask, or a data block related to a completely different expiration.

When a trade is involved with a message it appears before other blocks in the message. The M_3 message ID above illustrates a trade event and then a book update, which are denoted by Update IDs, U_{3a} and U_{3b}, respectively. For U_{3a}, an aggressive seller executes 5 contracts from the book at the best bid price of 50. The book updates (U_{3b}) in the same message ID (M_3) to show that depth is now 25 contracts derived from 4 orders. When the market is very active, the message ID may only contain trade blocks as the processing protocols call for this information to be transmitted quickly to participants. Then, the book update block will follow in a second message with the same message ID.

To rebuild the book for our sample, we select the most actively traded expiration for each calendar date, the message information for levels 1 and 2 of that expiration, and the trade-block updates. We systematically read the sequence of messages to find the depth at the best- and second-best quotes for the bid and ask IDs on the book. The final rebuilt book is a data table with the following variables: Message ID, Update ID, Time, Second-best bid (B2), Best Bid (B1), Best ask (A1), Second-best ask (A2), Depth at B2 (B2q), Depth at B1 (B1q), Depth at A1 (A1q) and Depth at A2 (A2q). Each observation captures some change in these variables. Most rows in this table show only one change of the four depth variables. Between the messages and time stamps, the book is static.

To distinguish between book changes due to trades and those due to cancels, we examine the same-side book and trade updates in message/time order. If there is a change in the quote and the previous depth can be accounted for by the quantity noted in the trade block (possibly many entries here), then the quote change is due to a trade. If the depth decreases, and there are no trades in a trade block that account for the decrease, then the change is due to cancellations. For example, if there is an aggressive trade for one contract (say), the message flow will show a trade at that price level then the book update shows a decrease in quantity. A cancel will show up as a book update with a decrease in depth, but no trade block.

Iceberg orders require another type of adjustment as they do not appear on the book until the displayed quantity executes to signal a draw of more contracts. In the normal course of trading, the execution of an order with additional hidden quantity will simply add to the depth at that quote. But in some cases—specifically, the quote flips we are focused on—a hidden order will appear when the trade block has exhausted previously known depth, but the update block shows no change in the best quotes and continues to show depth at those quotes. For these cases, we enter the new known depth size on the book and continue processing messages.

The rebuilt book simplifies the sequence of book updates. The message data often includes multiple update IDs, one for the message and one for the book updates contained in each message. Because these book updates represent a single action and related changes, we keep the last book update ID for each message knowing that all message-specific updates are now included here. For example, a modification moving a resting limit order from the second-best bid quote to the best bid quote will have a book update reducing depth at B2 and increasing depth at B1. The rebuilt book table shows a single row with both changes if only the bid side changed. This approach and our focus on the two best quote levels results in a meaningful reduction in the number of book updates and processing time.

The rebuilt book provides the bid-ask spread (BAS) for each update. The BAS is used to help identify quote change events. Specifically, we search for all cases in which there are changes in either of the best quotes and/or changes in the BAS. For example, assume the market starts with a 1-tick BAS. There can be two different ways the best quotes will change. One option is for the best ask or bid quote to be removed, either from trades or cancellations, resulting in a BAS greater than 1-tick. The second option is for the best quotes to move up or down in an immediate similar shift, which will not change the BAS. We refer to this second option as an "instant" quote flip and we find that most these instant flips are related to a 1-tick change in the best quotes. Because the BAS is 1-tick before and after the flip, we consider this a single quote event. For each of these instant quote flip events, we know the best quotes prior and following the event and we know the event arrival time.

When the best quotes do not change simultaneously, there are other options to consider. If the BAS is becomes two ticks and a subsequent quote change on the opposite side arises to create a 1-tick BAS, we refer to this event as a "lagged" quote flip because the outcome of the event matches the 1-tick change from an instant quote flip. If the BAS spread is restored to one tick, but there is no change in the opposite-side quote, then we have a "hat" type event. We may also observe that the initial side quote continues to change directionally, either further increasing the BAS or changing contemporaneously with the opposite-side quote to maintain a 2-tick spread. Rarely, there is an opposite-side quote change that continues to widen the BAS because the opposite side quote moved in the opposite direction. These latter two types of events are grouped into an "other" category, which is a small subset of quote events. For example, the arrival of an aggressive order that removes more than one quote from the book resulting in a BAS larger than 2-ticks. Generally, all movements in the best quotes of more than 1-tick are placed in the other group. Our methods label the instant and lagged quote events for analysis with the directional, hat, and other type events grouped into the other category.

The instant and lagged quote events can be thought of as having a start and end as determined by the initial quote levels and BAS. Panel A in Figure 6 shows that these quote events may revert, confirm, or continue as trade proceeds. To label the end of these events, we determine which of them revert or continue. Those that revert are the focus of our analysis. The ending of those events that continue with the new quote levels is determined by the arrival of the next quote event, which is not one that indicates reversion or continuation. Most of these endings are due to “hat” events. We document information on the duration between start and end points for the instant-, lagged-, and hat-type events, and track whether trades or cancels caused these events.

Appendix A2
Model Details

This appendix adds details to the discussion of the quote flipping model developed in the main text.

A) Bounding the Fundamental Value.

Following the methods of Dahlström, et. al. (2017), we show the conditions under which the fundamental value is bounded by the best bid and ask prices after liquidity providers optimally place limit orders on the discrete price grid. Specifically, solving equation (8) for α and substituting into equation (7), then solving for X_t gives:

$$X_t = \theta(P_1 - \gamma) + (1 - \theta)(P_{-1} + \gamma),$$

where $\theta = \left(\vec{Q}_{-1}^{B} + 2\varphi\right)/\left(\vec{Q}_{-1}^{B} + 2\varphi + \vec{Q}_{1}^{A} + 2\varphi\right)$. Because the elements, $\left\{\vec{Q}_{1}^{A}, \vec{Q}_{-1}^{B}, \varphi\right\} \in \mathbb{R}^{+}$, then $0 \leq \theta \leq 1$, so $P_1 \geq X_t \geq P_{-1}$ if $\frac{1}{2}(P_1 - P_{-1}) \geq \gamma$. Thus, a transaction fee less than one-half the best bid-ask spread (or minimum tick size) is sufficient to guarantee that the current fundamental value is bounded by the best ask and bid prices.

B) Quote Flips that Improve the Market

An issue in the quote flipper model is whether the quote flip is justified by an information effect, which leads quote flippers to act because the predicted fundamental value is changed sufficiently in the direction of the flip. That is, quote flippers observe m_1, thus they can calculate $E[X_{t+1}|m_1] = X_t + \alpha(m_1 + E[m_2|m_1]) = X_t + \alpha m_1$, as the two draws are independent and a liquidity-demanding buyer or seller arises with equal probability. If $m_1 > 0$ and $\alpha m_1 > P_1 - X_t$ or $m_1 < 0$ and $\alpha m_1 < P_{-1} - X_t$, then the actions of quote flippers are not opportunistic per sé, but may be justified as a reaction to the expected effects of informed trading on the fundamental price. Although there is no way to separate this situation from cases in which the information effect is small, so the flip is a targeting strategy, our focus on reverting flips reduces the noise produced by this possibility.

C) Quote Flipper Competitive Solution

The quote flippers expected profit on the ask side in equation (9) simplifies to the expression (bid side is analogous):

$$\frac{max(\vec{Q}_1^A - m_1, 0)}{2}\left\{2(X_t + \alpha m_1 - P_1 - \gamma) - \left(X_t + \alpha\left(\varphi + \vec{Q}_2^A + \vec{Q}_1^A\right) - (P_2 - \gamma)\right)e^{-\frac{\vec{Q}_2^A + \vec{Q}_1^A - m_1}{\varphi}}\right\}$$

The first term implies that expected profits are zero when $m_1 \geq \vec{Q}_1^A$ because there are no orders on the book to flip at the previous best ask price. The second term in brackets (denoted by *g(•)*) then determines when expected profits are driven to zero by competitors. Competitors enter to remove orders whenever m_1 is such that *g(•)* > 0. As there is no explicit solution for the value of m_1 that sets this term equal to zero, we simulated its effects in Figures 3 and 4. These simulations show that under a normal parameterization, expected profits are increasing as m_1 increases sequentially to fill the first draw from the liquidity demander's distribution until a maximum is reached. As the first term, $\frac{max(\vec{Q}_1^A - m_1, 0)}{2}$, decreases with m_1 the second term must be increasing until expected profits reach a maximum at m_1^*. Thus,

$$\frac{\partial g(m_1)}{\partial m_1} = 2\alpha - \frac{1}{\varphi}\left(X_t + \alpha\left(\varphi + \vec{Q}_2^A + \vec{Q}_1^A\right) - (P_2 - \gamma)\right)e^{-\frac{\vec{Q}_2^A + \vec{Q}_1^A - m_1}{\varphi}} > 0 \;\; if \; m_1 \leq m_1^*$$

The competitive solution flips more contracts, $(\vec{Q}_1^A - m_1)$, than the profit-maximizing solution. Thus, the sign holds at the competitive solution. The implicit function theorem shows how the competitive solution is affected by changes in alpha or phi. Specifically, $sign\left[\frac{dm_1^C}{d\alpha}\right] = -sign\left[\frac{\partial g(\alpha; m_1^C)}{\partial \alpha}\right] / sign\left[\frac{\partial g(m_1; m_1^C)}{\partial m_1}\right]$ and $sign\left[\frac{dm_1^C}{d\varphi}\right] = -sign\left[\frac{\partial g(\varphi; m_1^C)}{\partial \varphi}\right] / sign\left[\frac{\partial g(m_1; m_1^C)}{\partial m_1}\right]$, where the function *g(•)* is evaluated in the neighborhood of the competitive solution. These partial derivatives are as follows:

$$\frac{\partial g(\alpha; m_1^c)}{\partial \alpha} = 2m_1^c - \left(\varphi + \vec{Q}_2^A + \vec{Q}_1^A\right)e^{-\frac{\vec{Q}_2^A + \vec{Q}_1^A - m_1^c}{\varphi}}$$

$$\frac{\partial g(\varphi; m_1^c)}{\partial \varphi} = -\left\{\alpha + \frac{1}{\varphi^2}\left(\vec{Q}_2^A + \vec{Q}_1^A - m_1^c\right)\left(X_t + \alpha\left(\varphi + \vec{Q}_2^A + \vec{Q}_1^A\right) - (P_2 - \gamma)\right)\right\}e^{-\frac{\vec{Q}_2^A + \vec{Q}_1^A - m_1^c}{\varphi}}$$

Discussion: From the competitive solution above, $2(X_t + \alpha m_1^c - P_1 - \gamma) = \left(X_t + \alpha\left(\varphi + \vec{Q}_2^A + \vec{Q}_1^A\right) - (P_2 - \gamma)\right)e^{-\frac{\vec{Q}_2^A + \vec{Q}_1^A - m_1^c}{\varphi}}$. Multiplying $\frac{\partial g(\alpha; m_1^c)}{\partial \alpha}$ by α and re-arranging terms from the competitive solution gives, $2\alpha m_1^c - \alpha\left(\varphi + \vec{Q}_2^A + \vec{Q}_1^A\right)e^{-\frac{\vec{Q}_2^A + \vec{Q}_1^A - m_1^c}{\varphi}} = (X_t - P_2 + \gamma))e^{-\frac{\vec{Q}_2^A + \vec{Q}_1^A - m_1^c}{\varphi}} - 2(X_t - P_1 - \gamma)$. Thus, the sign of $\frac{dm_1^c}{d\alpha}$ is determined by whether $(X_t - P_2 + \gamma))e^{-\frac{\vec{Q}_2^A + \vec{Q}_1^A - m_1^c}{\varphi}}$ is greater or less than $2(X_t - P_1 - \gamma)$. As this first term has *de minimis* value, the sign is determined by $-2(X_t - P_1 - \gamma)$, which is positive for values of gamma such that $P_1 \geq X_t$ as discussed above. Thus, $sign\left[\frac{dm_1^c}{d\alpha}\right] < 0$, which is consistent with Figure 3 and also holds at the parameter values assumed in our simulations. The sign of the phi effect depends on the derivative, $\frac{\partial g(\varphi; m_1^c)}{\partial \varphi}$, shown above. Because this term is weighted by $e^{-\frac{\vec{Q}_2^A + \vec{Q}_1^A - m_1^c}{\varphi}}$ it also takes on *de minimis* values, which is why the competitive solutions in Figure 4 all appear to intersect at the same value of m_1. At the parameterizations in these simulations, the $sign\left[\frac{dm_1^c}{d\varphi}\right] > 0$, but the effect is zero to four decimal places. Thus, the incentive effect on quote flippers is due to the greater expected profits that arise from changes in phi as shown in Figure 4.

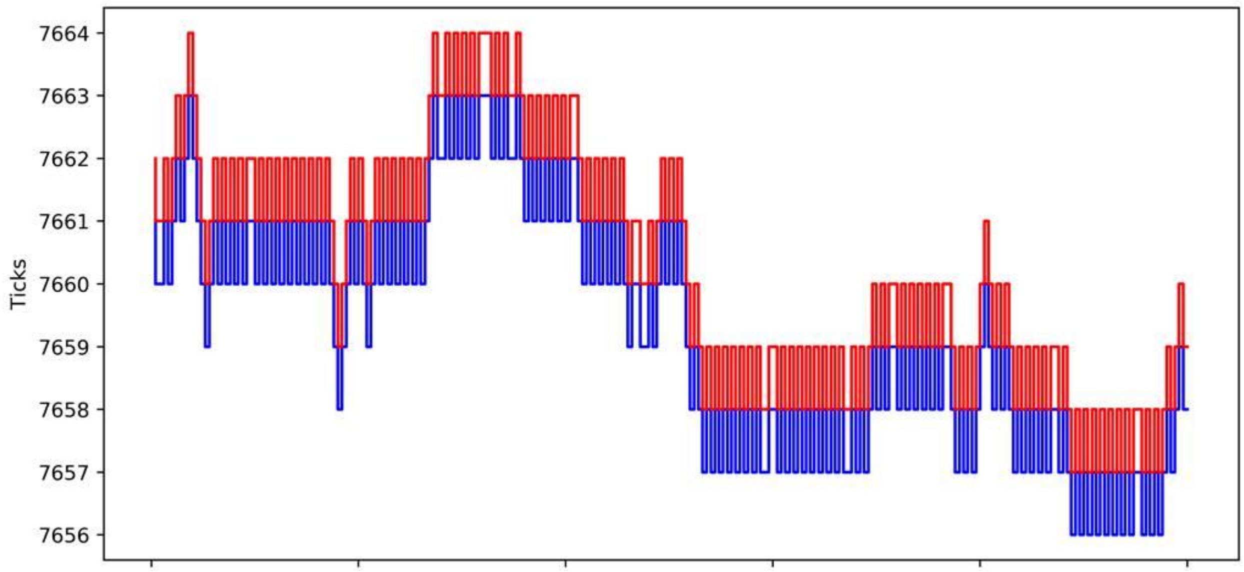

Figure 1 – Sample of Intraday Quote Changes for E-mini Futures.

This figure shows the path of 250 quote changes for the CME E-mini futures contract traded on August 6, 2014, beginning at approximately 10:00 A.M (E.T.), displayed in event time. The vertical axis measures price ticks. The sample is created from a rebuilt E-mini futures limit order book, keeping only observations with an observed change in the best quotes or trades at the best quotes. The step plot displays the best ask (red) and best bid (blue). The occasional gaps between changes in the best quotes result when there are trades that do not result in an immediate change in one or both of the best quotes.

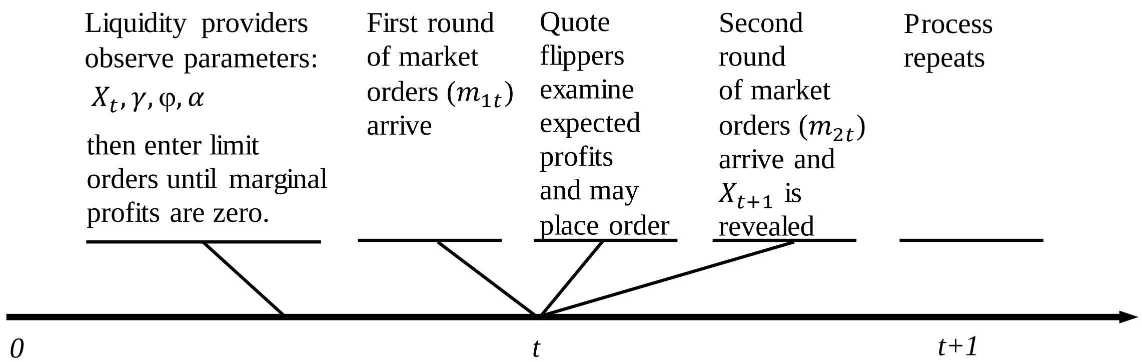

Figure 2 – Timeline of events.
The quote flipper model begins when liquidity providers place quotes on the order book up to the point at which marginal profits are zero. Market order traders then arrive at period *t*, some of whom may be informed. After the first round of market orders, quote flippers examine expected profits and may place orders to deplete the remaining depth, possibly leaving an open order that changes the ask to a new, best bid or vice versa. The quote flippers then place an exit order equal to the size traded at the new, best quote. The second round of market orders arrive, and the quote flippers' order executes or is left as inventory when the next period's true fundamental price is revealed. The process then repeats.

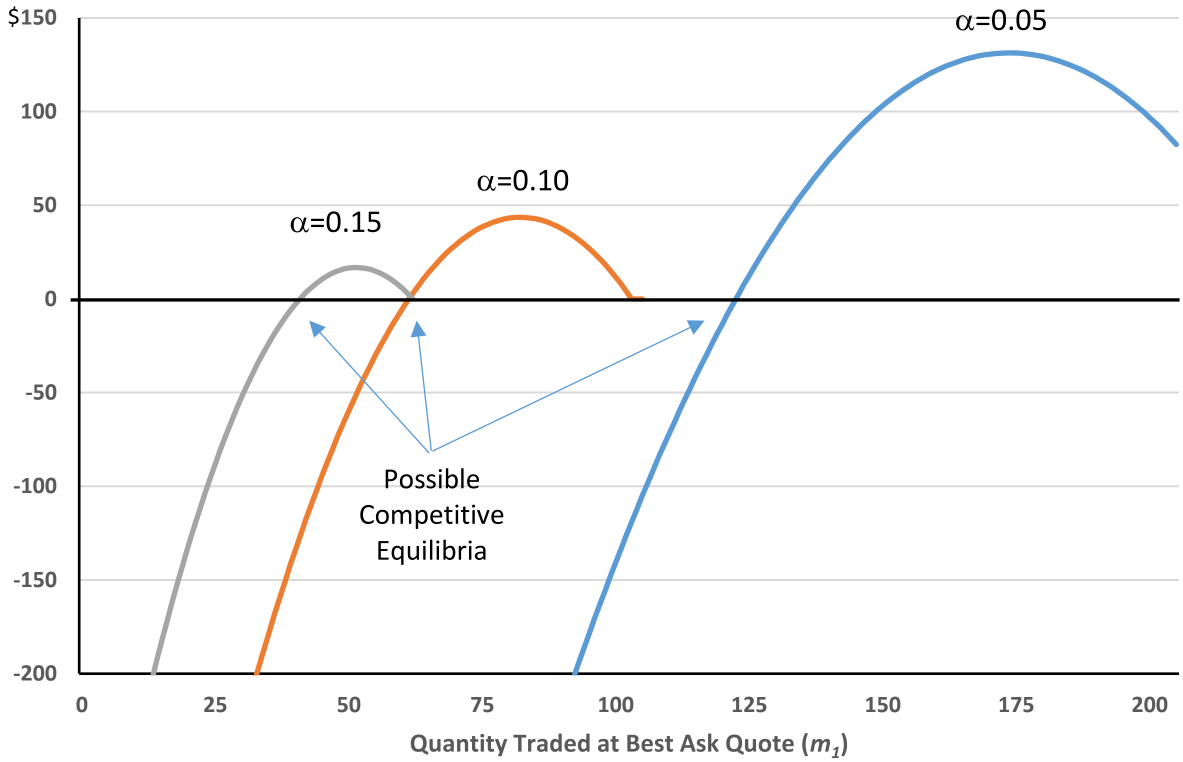

Figure 3 – Information/Adverse Selection Effects on Expected Profits for Quote Flippers
This figure shows how quote flippers' expected profits change when the information/adverse selection effect (α) decreases. Expected profits are computed for flipping the quantity, $\vec{Q}_1 - m_1$, on the ask side of the order book. The model is parameterized as follows: P_1=\$12.50, P_2=\$25.00, X_t=\$6.25, φ=10, γ=0. Equations (7) and (8) are used to determine the optimal ask depth at P_1 and P_2, respectively. The plotted lines terminate when $\vec{Q}_1 - m_1 = 0$, which adjusts based on how the adverse selection effect alters the optimal depth ($\vec{Q}_1$) at the best ask. The possible competitive solutions are labelled for each curve.

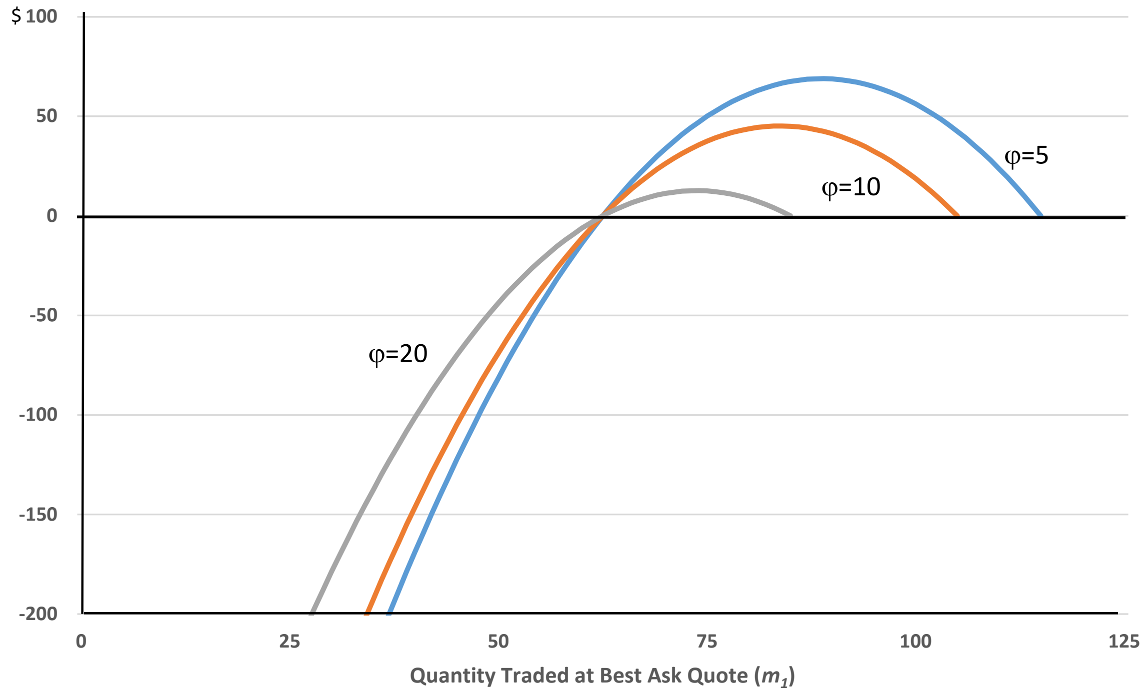

Figure 4 – Effects of Average Order Flows on Expected Profits of Flippers
This figure shows how quote flippers' expected profits change when the distributional parameter (φ) changes. This parameter equals expected order flow for the single draw exponential distribution. Expected profits are computed for flipping the quantity, $\vec{Q}_1 - m_1$, on the ask side of the order book. The model is parameterized as follows: P_1=\$12.50, P_2=\$25.00, X_t=\$6.25, α=0.10, γ=0. Equations (7) and (8) are used to determine the optimal ask depth at P_1 and P_2, respectively. The plotted lines terminate when $\vec{Q}_1 - m_1 = 0$, which adjusts based on how the distributional parameter alters the optimal depth ($\vec{Q}_1$) at the best ask.

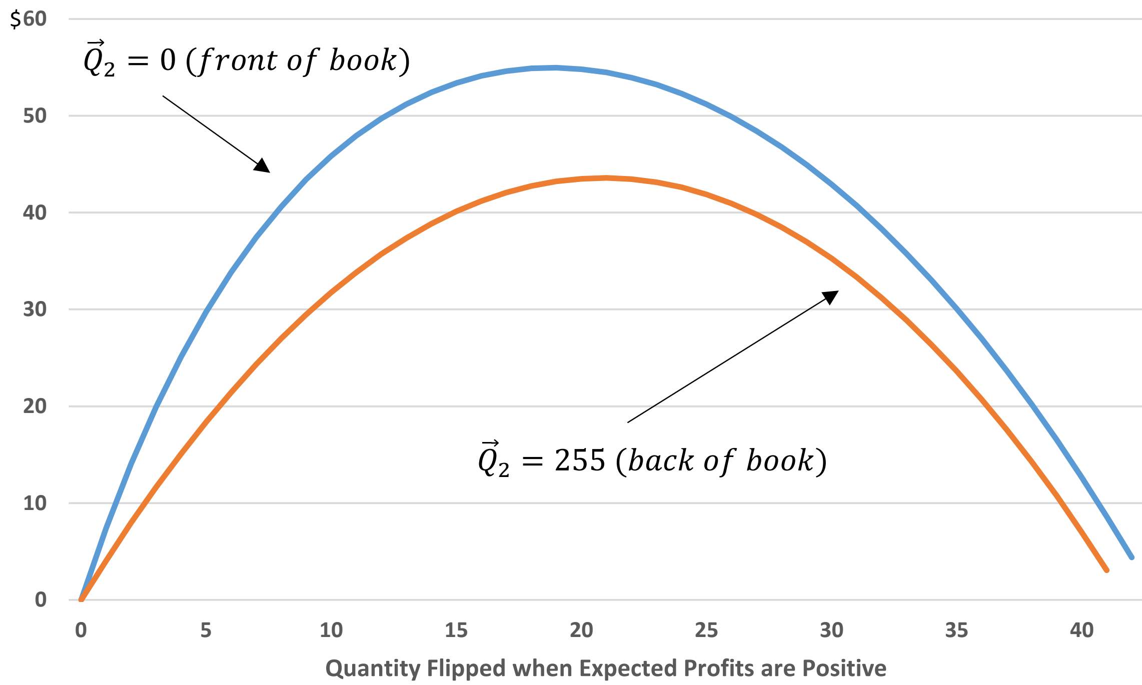

Figure 5 – Depth Effects at Next-Best Quote
This figure shows how quote flippers' expected profits change when the depth at the next-best quote diminishes and the quantity flipped changes. The top curve represents expected profits when the quote flippers' exit quantity is at the front of the book when placed at the next-best ask quote. The bottom curve represents expected profits when the quote flippers' exit quantity is placed at the back of the book. The depth in the latter case is determined by Equation (8). The model is parameterized as follows: P_1=$12.50, P_2=$25.00, X_t=$6.25, α=0.10, φ=10, γ=0. The graph only shows the region of positive profits for these two simulations.

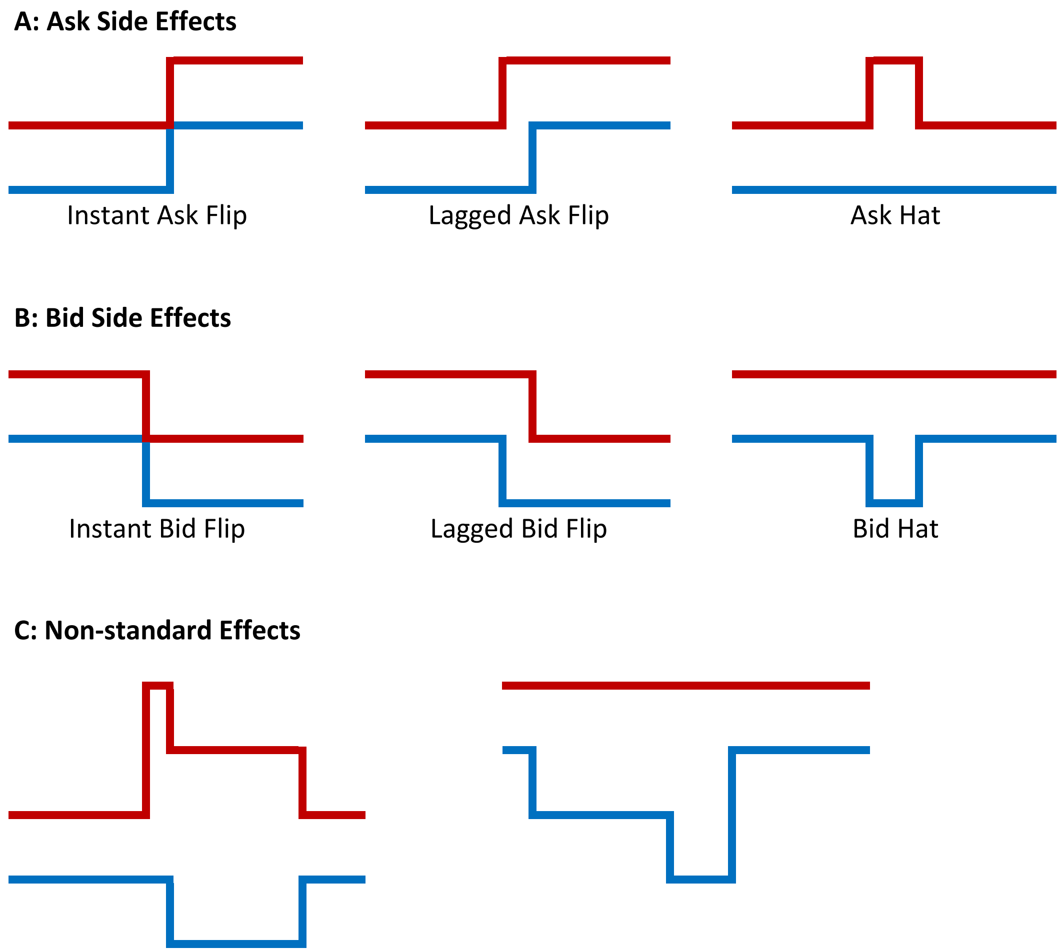

Figure 6 – Quote Change Patterns

Panel A and B shows the three primary event types: instant flip, lagged flip and the hat pattern for the ask- and bid-sides, respectively. For these patterns, the market starts and ends with a bid-ask spread of 1-tick. Instant flip events occur when a quote is instantaneously flipped by an aggressive order extracting all available depth and leaving an unfilled portion at the same quote level. Lagged flip events arise when depth is exhausted by trades or cancellations and the spread temporarily changes to two ticks. A hat event arises when only one side of the market is affected by the quote change, which is reversed is a short time. Panel C shows two types of non-standard flips, which include quote movements larger than 1-tick or more than two quote changes that result in a bid-ask spread larger than 2-ticks.

Table 1
Quote Change Event Patterns

This table documents the frequency of different quote change events based on the initial direction of the price change and the type of quote flip (if any). A "Down" ("Up") quote movement implies that both the bid and ask price decreases (increases). If the movement is "non-standard" it means that only one side of the quote changed in the direction indicated (e.g., a "Hat" or "Step" pattern). The initial flip type denotes whether both quotes changed in the same instant of time (1 millisecond) or whether there was a lag in the response of one side of the quote spread. If the initial movement is "down", then the lag is on the ask side; for "Up" it is on the bid side. The table also documents how the quote change event ends based on the next type of quote movement. The duration of time between these events is measured in seconds/milliseconds and the average, standard deviation, first and third quartiles, and median are reported.

						Duration of Initial Event (Seconds)				
Initial Quote Movement	**Initial Flip Type**	**Next Quote Movement**	**Next flip Type**	**Event Count**	**Percent**	**Mean**	**StdDev**	**Q1**	**Median**	**Q3**
	Lag	Down	Lag	1,244	0.7%	9.49	16.28	1.64	4.46	10.81
	Lag	Down	Instant	4,164	2.3%	11.55	25.59	1.97	5.11	11.88
	Instant	Down	Lag	3,303	1.8%	11.02	28.50	1.67	4.55	11.38
	Instant	Down	Instant	13,332	7.4%	11.47	21.93	1.96	5.03	12.07
DOWN	Lag	Non-standard	NA	6,796	3.8%	0.59	4.27	0.00	0.00	0.03
	Instant	Non-standard	NA	11,689	6.5%	0.77	5.64	0.00	0.00	0.02
	Lag	Up	Lag	13,774	7.6%	1.39	6.50	0.01	0.07	0.50
	Lag	Up	Instant	26,177	14.4%	2.32	9.30	0.00	0.06	1.03
	Instant	Up	Lag	26,214	14.5%	1.97	8.03	0.01	0.07	0.70
	Instant	Up	Instant	74,492	41.1%	2.57	10.76	0.00	0.01	1.01
	NA	DN	Lag	7,234	13.8%	0.78	4.81	0.00	0.00	0.09
NOT	NA	DN	Instant	11,275	21.5%	1.63	7.93	0.00	0.00	0.15
ASSIGNED	NA	Non-standard	NA	15,492	29.5%	0.09	1.06	0.00	0.00	0.00
	NA	UP	Lag	7,239	13.8%	0.74	4.71	0.00	0.00	0.10
	NA	UP	Instant	11,227	21.4%	1.54	7.78	0.00	0.00	0.19
	Lag	DN	Lag	13,781	7.6%	1.37	5.90	0.01	0.07	0.50
	Lag	DN	Instant	25,878	14.3%	2.31	9.38	0.00	0.06	1.03
	Instant	DN	Lag	26,605	14.7%	1.95	8.39	0.01	0.07	0.69
	Instant	DN	Instant	74,352	41.1%	2.73	11.98	0.00	0.01	1.01
UP	Lag	Non-standard	NA	6,746	3.7%	0.51	3.59	0.00	0.00	0.03
	Instant	Non-standard	NA	11,746	6.5%	0.79	5.36	0.00	0.00	0.02
	Lag	UP	Lag	1,152	0.6%	11.57	23.02	1.83	4.86	12.24
	Lag	UP	Instant	4,029	2.2%	11.04	19.95	2.11	5.15	12.02
	Instant	UP	Lag	3,265	1.8%	11.41	23.47	1.92	4.80	11.92
	Instant	UP	Instant	13,475	7.4%	12.27	23.32	2.06	5.27	12.88

Table 2
Volumes During Isolated and Grouped Quote Events

The table shows E-mini contract volume oberved at the best quotes, second-best quotes, and at quotes away in the book during three types of windows. Isolated events are identified by having a separation of at least 20 milliseconds (ms) from neighboring events; all non-isolated quote events are placed into grouped sets.Volume during isolated event windows is calculated for the 10ms before and after an isolated event. Using the 20ms range guarantees that there are no overlapping windows to create a double counting of volume. Grouped event windows are defined as the interval containing all nearby events with 10ms before the first and 10ms after the last event within each group. After carving out windows for isolated and grouped events, all of the remaining activity is labeled as non-events. The total percent of volume and the percent of clock time for each window is also shown.

	Quotes				
Window	Best	Second Best	Deep in the Book	Share of Total Volume	Share of Clock Time in Sample
Isolated Events	21,085,876	615,003	24,728	41.2%	0.24%
Grouped Events	11,946,022	501,530	98,215	23.8%	6.91%
Non-Events	18,404,554			34.9%	92.72%

Table 3
Volume of Trading Before and After an Isolated Quote Event

This table reports the E-mini contract volume in 2 millisecond (ms) intervals prior to and after an isolated quote change. All isolated events have a separation of at least 20 milliseconds (ms) from any neighboring events. Isolated event windows are defined for the 10ms before and after the event. Given the 20ms interval, these 10ms before and after windows guarantee there is no overlapping volume counted in the table. Volume is shown at the best quotes as in Table 2 with the information divided into those quote changes caused by trades or cancellations. These data are further divided by those that reverted to the previous level and quote events that continued directionally for at least one additional tick change in price. Remaining isolated quote changes are placed in the "other quote events" category (e.g., "hat" shaped patterns). The share of volume is shown by time interval relative to the total contract volume at the best quotes observed for isolated quote events (21,085,876) and the total sample volume (51,436,452).

	Best Quotes - Contract Volume					Share of Isolated Volume				Share of Total Volume			
						Quote Reverts		Quote Continues		Quote Reverts		Quote Continues	
Interval (20 ms)	Trade-Caused Quote Event Reverts	Trade-Caused Quote Event Continues Directionally	Cancel-Caused Quote Event Reverts	Cancel-Caused Quote Event Continues Directionally	Other Quote Events	Trade-Caused Change	Cancel-Caused Change	Trade-Caused Change	Cancel-Caused Change	Trade-Caused Change	Cancel-Caused Change	Trade-Caused Change	Cancel-Caused Change
-10 ms	81,774	38,747	8,102	4,097	4,783	0.4%	0.0%	0.2%	0.0%	0.2%	0.0%	0.1%	0.0%
-8 ms	183,230	90,010	20,774	8,959	15,073	0.9%	0.1%	0.4%	0.0%	0.4%	0.0%	0.2%	0.0%
-6 ms	584,980	264,102	59,821	26,423	43,359	2.8%	0.3%	1.3%	0.1%	1.1%	0.1%	0.5%	0.1%
-4 ms	1,798,553	754,435	180,733	83,129	138,271	8.5%	0.9%	3.6%	0.4%	3.5%	0.4%	1.5%	0.2%
-2 ms	3,194,313	1,278,081	823,877	357,724	244,347	15.1%	3.9%	6.1%	1.7%	6.2%	1.6%	2.5%	0.7%
Event	6,100,399	3,165,450			374,656	28.9%	0.0%	15.0%	0.0%	11.9%	0.0%	6.2%	0.0%
+2 ms	108,870	51,353	10,776	4,293	16,248	0.5%	0.1%	0.2%	0.0%	0.2%	0.0%	0.1%	0.0%
+4 ms	116,601	58,744	12,928	5,735	12,191	0.6%	0.1%	0.3%	0.0%	0.2%	0.0%	0.1%	0.0%
+6 ms	185,900	85,859	17,117	5,681	17,693	0.9%	0.1%	0.4%	0.0%	0.4%	0.0%	0.2%	0.0%
+8 ms	152,629	77,923	15,287	6,801	17,779	0.7%	0.1%	0.4%	0.0%	0.3%	0.0%	0.2%	0.0%
+10 ms	89,316	57,453	10,124	5,131	15,242	0.4%	0.0%	0.3%	0.0%	0.2%	0.0%	0.1%	0.0%

Table 4
Interval Characteristics

The E-mini futures data are partitioned into 5- and 60-second intervals during active trading hours, which are between 8:00AM and 4:30PM (ET) in the E-mini futures market. The dataset covers 42 trading days between August 1st and September 30th in 2014. This table documents the characteristics of these data during the intervals where five or more contracts are traded. The variables are in groups: volume information, quote flip counts, cancelation counts, exit trades, and explanatory variables used in the count regressions. The explanatory variables are designed to measure parameters in the theoretical model. These include an estimate of Phi, equal to the average trade size in an interval; proxy for alpha, computed as the maximum quote range in the previous interval divided by (lagged) volume; and variables measuring the beginning depth from the orders on the book. Best and second-best beginning ask and bid depths are shown for these measures.

	5-Second Intervals				60-Second Intervals			
Variables	**Interval Count**	**Mean**	**Std. Dev.**	**Median**	**Interval Count**	**Mean**	**Std. Dev.**	**Median**
Volume Summary								
Total Volume	133,810	369.5	570.9	225.0	20,672	2548.8	3094.5	1663.5
Buyer-Initiated Volume	133,810	184.0	323.1	85.0	20,672	1268.6	1584.6	805.0
Seller-Initiated Volume	133,810	185.6	332.9	87.0	20,672	1280.2	1670.6	797.0
Quote Flip Counts								
Instant Bid Flip	133,810	0.95	1.85	0.00	20,672	6.26	9.58	4.00
Lagged Bid Flip	133,810	0.39	0.98	0.00	20,672	2.53	4.67	1.00
Not Assigned Bid Chg.	133,810	0.20	1.04	0.00	20,672	1.28	4.16	0.00
Instant Ask Flip	133,810	0.95	1.83	0.00	20,672	6.28	9.47	4.00
Lagged Ask Flip	133,810	0.38	0.97	0.00	20,672	2.50	4.63	1.00
Not Assigned Ask Chg.	133,810	0.19	0.95	0.00	20,672	1.27	3.61	0.00
Cancellation Counts								
Instant Bid Flip by Cancel	133,810	0.21	0.87	0.00	20,672	1.39	3.66	0.00
Lagged Bid Flip by Cancel	133,810	0.10	0.45	0.00	20,672	0.62	1.82	0.00
Instant Ask Flip by Cancel	133,810	0.21	0.86	0.00	20,672	1.39	3.59	0.00
Lagged Ask Flip by Cancel	133,810	0.10	0.45	0.00	20,672	0.62	1.82	0.00
Exit Trades								
All Trade Counts	133,810	96.5	112.5	67.0	20,672	671.8	697.0	457.0
New Bid trades a/ Instant Bid Flip	33,848	59.0	76.2	35.0	16,993	163.4	152.5	120.0
New Bid Trades a/ Lagged Bid Flip	13,022	45.5	57.6	21.0	8,188	75.5	89.1	53.0
New Ask Trades a/ Instant Ask Flip	33,929	58.8	76.1	36.0	14,593	143.1	170.2	94.0
New Ask Trades a/ Lagged Ask Flip	12,688	44.5	58.7	20.0	8,073	73.1	87.7	50.0
Explanatory variables								
Max. Quote ΔP/ Volume (α)	133,789	0.0175	0.0252	0.0078	20,665	0.0030	0.0051	0.0017
Estimated Phi (φ)	133,810	3.77	2.75	3.25	20,672	3.69	1.41	3.43
Best Ask Beginning Depth	133,810	487.5	595.7	320.0	20,672	454.5	526.9	313.0
Best Bid Beginning Depth	133,810	482.3	598.2	317.0	20,672	446.2	522.8	308.0
2nd Best Ask Beginning Depth	133,810	1008.1	697.5	854.0	20,672	976.7	626.6	869.0
2nd Best Bid Beginning Depth	133,810	1011.6	737.7	846.0	20,672	977.4	666.5	855.0

Table 5
Count Regressions: 5-Second Intervals

The explanatory variables found in Table 4 are included in count regressions to explain the number of flip events found in 5-second interval data. Flips caused by cancellations are excluded from these counts. The coefficients are estimated using Maximum Likelihood techniques under the assumption that the counts follow a negative binomial distribution. For comparision of mean significance, the Generalized Estimating Equation (GEE) method is also used to cluster errors over trading days in the sample. Panel A shows results for best bid flips and Panel B shows results for best ask flips. The variables used to estimate alpha and phi in the theoretical model are lagged to avoid endogeneity in the current interval. The depth measure reports the depth at the beginning of the current interval. Daily fixed effects are included in the maximum likelihood models. Wald chi-square statistics are used to compute p-values for the estimated coefficients, which are shown below each coefficient. An estimate of the variance dispersion parameter is also shown with 95% confidence intervals below the estimate. Goodness-of-fit statistics are shown at the bottom of the table for maximum likelihood estimates. These are the Akaike's information criterion (AIC) and Cameron and Trivedi (2013) pseudo R-squared measures. The sample size is 130,604 for both bid and ask count regressions.

	Dependent. Variable: Count of Instant Flips				Dependent Variable: Count of Lagged Flips			
Variables	(1)	(2)	(3)	GEE	(4)	(5)	(6)	GEE
Panel A: Flips of the Best Bid								
Intercept	-0.086	0.340	0.076	0.035	-0.889	-0.113	-0.395	-0.495
	<0.001	<0.001	<0.001	<0.001	<0.001	0.001	<0.001	<0.001
Lag[Max. Quote ΔP/Volume (α)]		-0.663	-0.237	-0.284		-0.877	-0.405	-0.432
		<0.001	<0.001	<0.001		<0.001	<0.001	<0.001
lag[Estimated Phi (φ)]		-0.004	-0.011	-0.010		-0.019	-0.028	-0.028
		0.046	<0.001	<0.001		<0.001	<0.001	<0.001
Best Bid Beginning Depth		-0.0005	-0.0005	-0.0005		-0.0007	-0.0007	-0.0008
		<0.001	<0.001	<0.001		<0.001	<0.001	<0.001
2nd Best Bid Beginning Depth		-0.0003	-0.0002	-0.0003		-0.0006	-0.0005	-0.0006
		<0.001	<0.001	<0.001		<0.001	<0.001	<0.001
Lagged Total Volume			0.0000	0.0000			0.0000	0.0000
			0.152	0.262			0.11	0.111
Lagged Trade Count			0.0021	0.0021			0.0023	0.0024
			<0.001	<0.001			<0.001	<0.001
Dispersion	0.92	0.80	0.69		1.51	1.21	1.06	
	(0.90-0.94)	(0.78-0.82)	(0.67-0.71)		(1.46-1.56)	(1.17-1.25)	(1.02-1.10)	
Daily Fixed Effects	Yes	Yes	Yes	No	Yes	Yes	Yes	No
AIC	315355	309319	304436		190269	184679	182143	
Log Likelihood	-157635	-154612	-152169		-95091	-92293	-91022	
Pseudo R-Squared	3.0%	8.0%	12.0%		3.0%	9.5%	12.5%	

Table 5 (continued)
Count Regressions: 5-Second Intervals

	Dependent. Variable: Count of (net) Instant Flips				Dependent Variable: Count of (net) Lagged Flips			
Variables	(1)	(2)	(3)	GEE	(4)	(5)	(6)	GEE
Panel B: Flips of the Best Ask								
Intercept	-0.118	0.318	0.064	0.057	-0.838	-0.021	-0.325	-0.513
	<0.001	<0.001	0.008	0.029	<0.001	0.541	<0.001	<0.001
α: Lag[Max. Quote ΔP/Volume]		-0.675	-0.258	-0.303		-0.964	-0.465	-0.484
		<0.001	<0.001	<0.001		<0.001	<0.001	<0.001
φ: Lag[Avg. Trade Size]		-0.004	-0.012	-0.010		-0.019	-0.024	-0.023
		0.035	<0.001	0.001		<0.001	<0.001	<0.001
Best Ask Beginning Depth		-0.0006	-0.0006	-0.0006		-0.0007	-0.0007	-0.0008
		<0.001	<0.001	<0.001		<0.001	<0.001	<0.001
2nd Best Ask Beginning Depth		-0.0003	-0.0002	-0.0003		-0.0006	-0.0006	-0.0006
		<0.001	<0.001	<0.001		<0.001	<0.001	<0.001
Lag[Total Volume]			0.0000	0.0000			-0.0001	-0.0001
			0.012	0.002			<0.001	0.001
Lag[Trade Count]			0.0021	0.002			0.0026	0.0027
			<0.001	<0.001			<0.001	<0.001
Dispersion	0.89	0.77	0.67		1.56	1.23	1.06	
	(0.88-0.92)	(0.75-0.79)	(0.65-0.68)		(1.50-1.60)	(1.18-1.27)	(1.03-1.11)	
Daily Fixed Effects	Yes	Yes	Yes	No	Yes	Yes	Yes	No
AIC	315595	309220	304446		188997	183206	180705	
Log Likelihood	-157755	-154563	-152174		-94456	-91556	-90303	
Pseudo R-Squared	3.1%	8.3%	12.2%		2.7%	9.5%	12.5%	

Table 6
Count Regressions: 60-Second Intervals

The explanatory variables found in Table 4 are included in count regressions to explain the number of flip events in 60-second interval data. Flips caused by cancellations are excluded from these counts. The coefficients are estimated using Maximum Likelihood techniques under the assumption that the counts follow a negative binomial distribution. For comparision of mean significance, the Generalized Estimating Equation (GEE) method is also used to cluster errors over trading days in the sample. Panel A shows results for best bid flips and Panel B shows results for best ask flips. The variables used to estimate alpha and phi in the theoretical model are lagged to avoid endogeneity in the current interval. The depth measure reports the depth at the beginning of the current interval. Daily fixed effects are included in the maximum likelihood models. Wald chi-square statistics are used to compute p-values for the estimated coefficients, which are shown below each coefficient. An estimate of the variance dispersion parameter is also shown with 95% confidence intervals below the estimate. Goodness-of-fit statistics are shown at the bottom of the table for maximum likelihood estimates. These are the Akaike's information criterion (AIC) and Cameron and Trivedi (2013) pseudo R-squared measures. The sample size is 20,623 (20,630) for bid (ask) count regressions.

	Dependent. Variable: Count of (net) Instant Flips				Dependent Variable: Count of (net) Lagged Flips			
Variables	(1)	(2)	(3)	GEE	(4)	(5)	(6)	GEE
Panel A: Flips of the Best Bid								
Intercept	1.994	2.630	1.580	1.326	1.144	2.240	1.399	1.204
	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
α: Lag[Max. Quote ΔP/Volume]		-69.920	-21.440	-23.520		-74.890	-47.740	-52.410
		<0.001	<0.001	0.041		<0.001	<0.001	<0.001
φ: Lag[Avg. Trade Size]		-0.072	-0.052	-0.030		-0.123	-0.132	-0.123
		<0.001	<0.001	0.145		<0.001	<0.001	<0.001
Best Bid Beginning Depth		-0.0002	-0.0002	-0.0002		-0.0002	-0.0003	-0.0003
		<0.001	<0.001	<0.001		<0.001	<0.001	<0.001
2nd Best Bid Beginning Depth		-0.0002	-0.0001	-0.0002		-0.0005	-0.0004	-0.0006
		<0.001	<0.001	<0.001		<0.001	<0.001	<0.001
Lag[Total Volume]			0.0000	0.0000			0.0000	0.0000
			<0.001	<0.001			<0.001	0.026
Lag[Trade Count]			0.0009	0.001			0.0008	0.0008
			<0.001	<0.001			<0.001	<0.001
Dispersion	0.91	0.75	0.53		1.13	0.82	0.64	
	(0.88-0.93)	(0.73-0.77)	(0.51-0.55)		(1.09-1.17)	(0.79-0.86)	(0.61-0.67)	
Daily Fixed Effects	Yes	Yes	Yes	No	Yes	Yes	Yes	No
AIC	107325	105497	100263		74877	72369	69459	
Log Likelihood	-53619	-52701	-50083		-37396	-36137	-34680	
Pseudo R-Squared	14.6%	22.5%	45.2%		12.7%	24.6%	38.2%	

Table 6 (continued)
Count Regressions: 60-Second Intervals

Variables	Dependent. Variable: Count of (net) Instant Flips (1)	(2)	(3)	GEE	Dependent Variable: Count of (net) Lagged Flips (4)	(5)	(6)	GEE
Panel B: Flips of the Best Ask								
Intercept	1.951	2.670	1.675	1.482	1.200	2.344	1.414	1.156
	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
α: Lag[Max. Quote ΔP/Volume]		-77.430	-30.708	-33.431		-75.980	-51.815	-56.163
		<0.001	<0.001	<0.001		<0.001	<0.001	<0.001
φ: Lag[Avg. Trade Size]		-0.092	-0.082	-0.065		-0.133	-0.122	-0.116
		<0.001	<0.001	<0.001		<0.001	<0.001	<0.001
Best Bid Beginning Depth		-0.0002	-0.0002	-0.0002		-0.0001	-0.0002	-0.0002
		<0.001	<0.001	<0.001		<0.001	<0.001	<0.001
2nd Best Bid Beginning Depth		-0.0002	-0.0001	-0.0002		-0.0006	-0.0004	-0.0006
		<0.001	<0.001	<0.001		<0.001	<0.001	<0.001
Lag[Total Volume]			0.0000	0.0000			-0.0001	-0.0001
			<0.001	<0.001			<0.001	<0.001
Lag[Trade Count]			0.0008	0.0009			0.0009	0.0008
			<0.001	<0.001			<0.001	<0.001
Dispersion	0.89	0.71	0.51		1.13	0.82	0.63	
	(0.87-0.92)	(0.69-0.73)	(0.49-0.53)		(1.09-1.17)	(0.79-0.85)	(0.61-0.66)	
Daily Fixed Effects	Yes	Yes	Yes	No	Yes	Yes	Yes	No
AIC	107491	105334	100220		74722	72160	69250	
Log Likelihood	-53703	-52620	-50061		-37318	-36033	-34576	
Pseudo R-Squared	14.7%	24.1%	46.2%		11.9%	24.0%	37.7%	